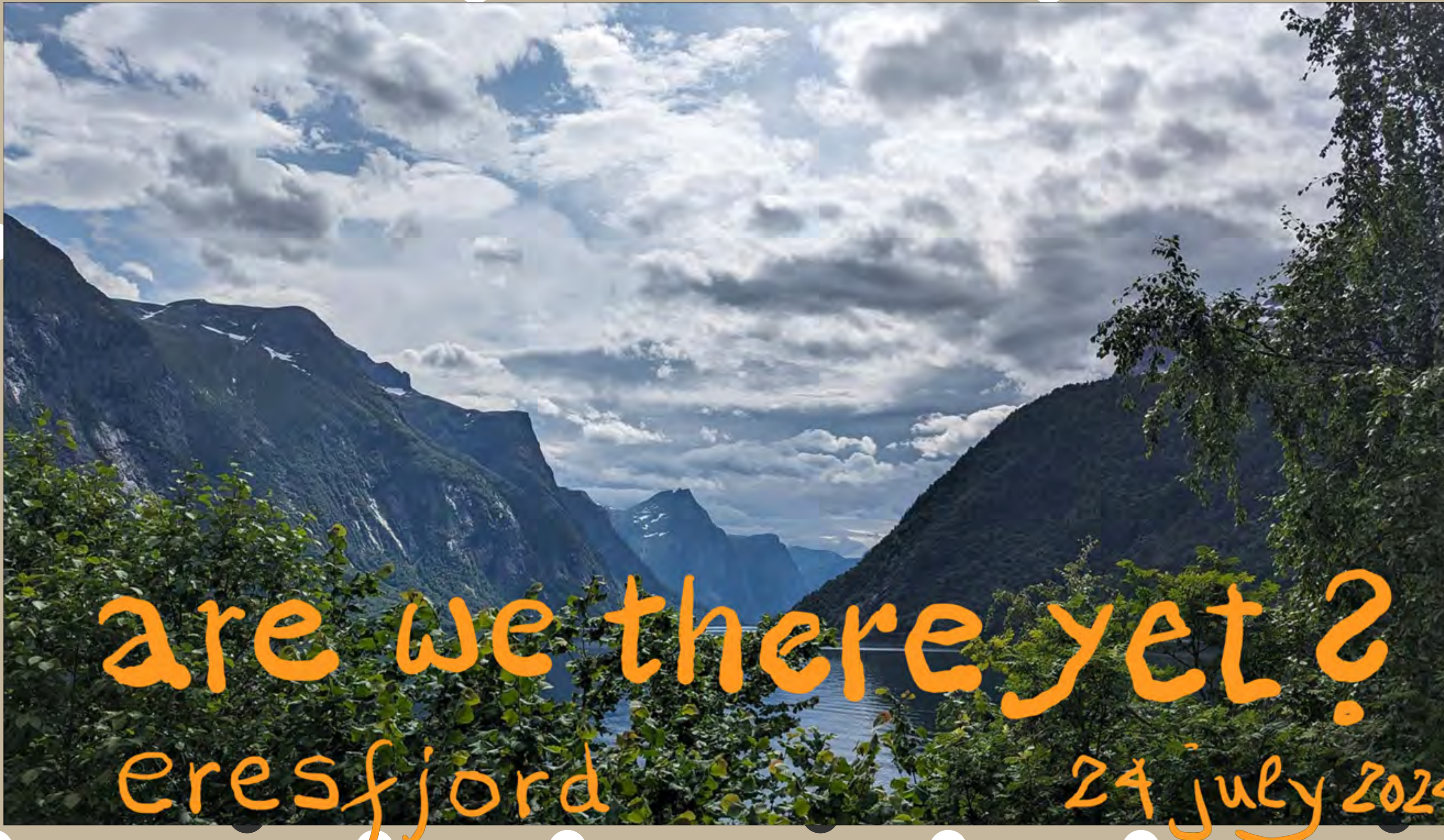
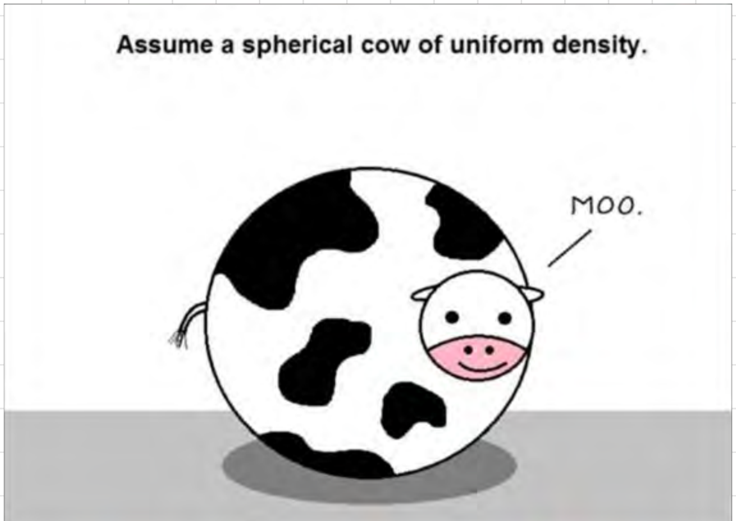

are we there yet?
eresfjord
24 july 2024

A physicist; real neuroscientist



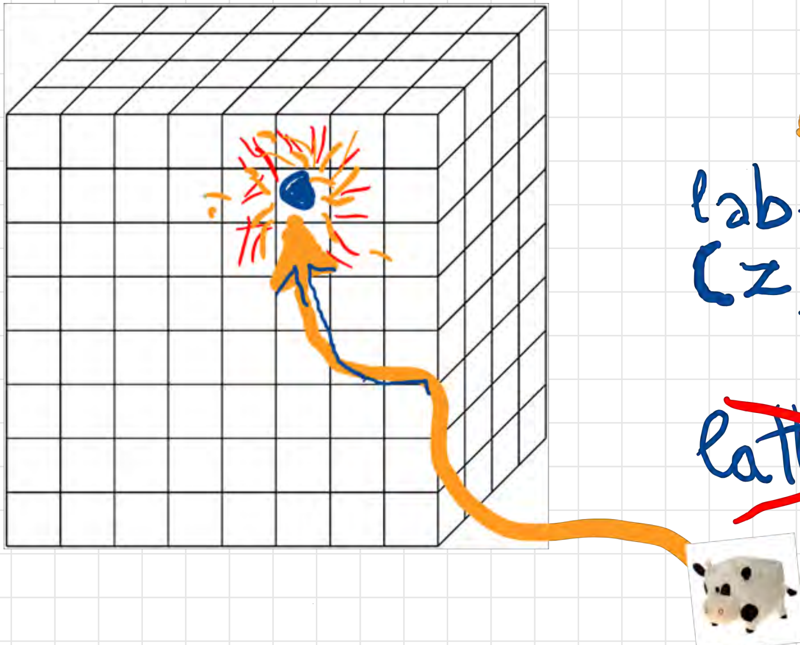
ha, ha, ha

realistically accurate model

?? !!!!

a "Spacetime ~~Lattice~~ Brain"

"position" time



label

$(z_1, z_2, z_3, \ldots, z_d)$ ← time

~~lattice site~~ neuron

"the floor", "the network", "architecture" neuron activity

location $(z_1, z_2, \dots, z_d)$

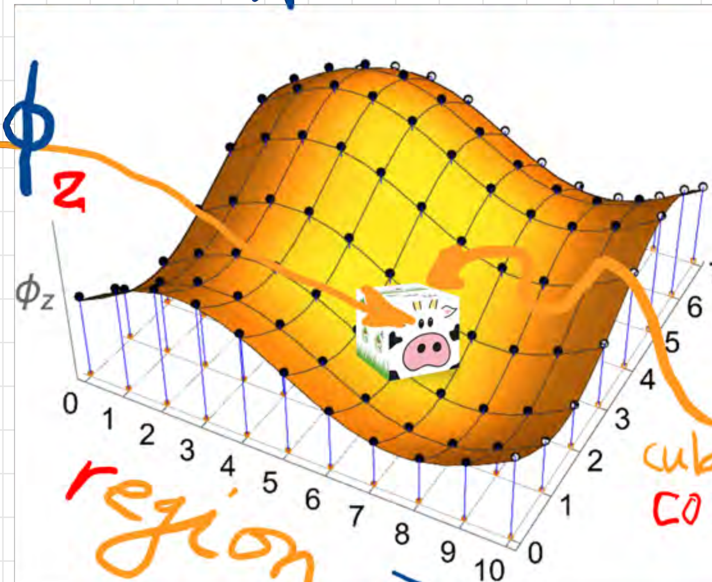
time

an episode

not a snapshot!

a state of "neurons"

ϕ_z



region $\mathbb{A}$

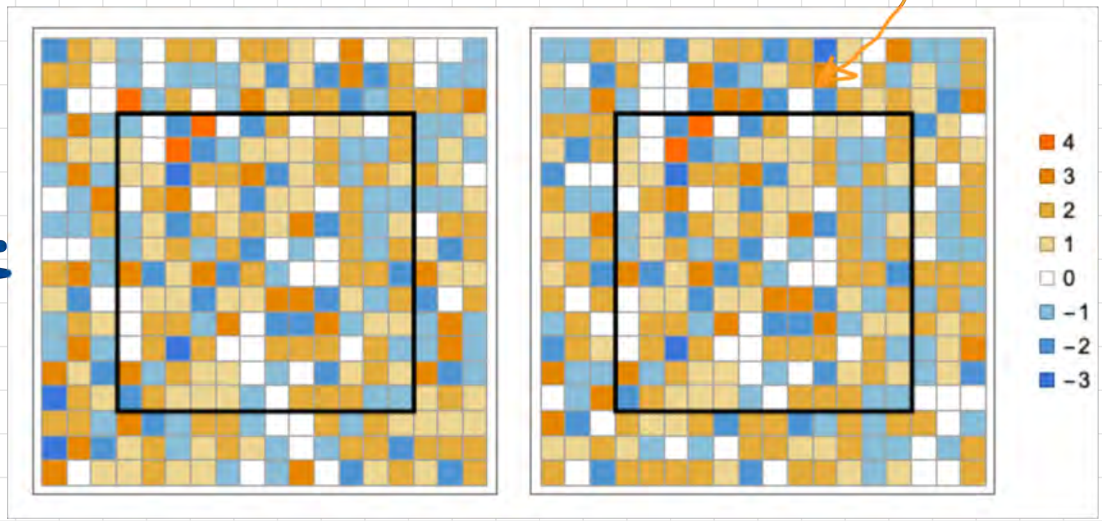
a cubic cow

$N_{\mathbb{A}}$-dimensional

This lecture:

Q. How close are

neuron state, color coded

$\Phi_1 =$ 

$= \Phi_2$

?

here is a stupid idea

$$d^2(\Phi_1, \Phi_2) = \sum_{z}^{A} (\phi_{1,z} - \phi_{2,z})^2$$

domain

neuron

"activity"

episode 1

another episode

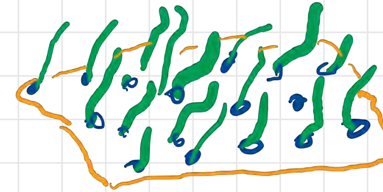
me $\approx (11.3\,\text{mV})^2$

you $= (\text{calc./potasin})^2$

are we close?, are we far???

Probability of a field configuration

$$p[\Phi] = \frac{1}{Z} e^{-S[\Phi]}$$



Λ

$$Z = \int d\Phi \, e^{-S[\Phi]} , \quad d\Phi = \prod_{z}^{\Lambda} d\phi_z$$

partition function

is determined by

THE LAW

$$\boxed{F[\Phi_c] = 0}$$

Z

allowed

upside down

forbudt!

every neuron z

$$\boxed{F[\Phi_c] = 0} \quad z$$

looks left, right
tomorrow, yesterday
introspects

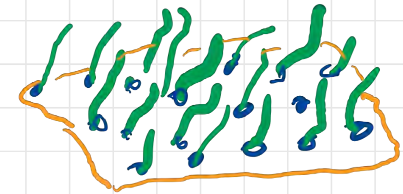
here
deterministic

might smear
with
noise
later

deterministic partition function

probability density (up to $\frac{1}{Z}$ factor)

$$p_Z[\Phi_c] = \delta_Z(F[\Phi_c])$$

$\Phi_c =$ 

state "c"

(a deterministic solution)

integrated; sum over all

$$Z = \sum_c \frac{1}{|\det J_c|}$$

orbit Jacobian

where did this determinant come from?

Dirac δ-function

$$\int_I dx\, \delta(x) = 1 \quad \text{if } x \in I$$

$$\int_I dx\, \delta(ax) = \frac{1}{a} \qquad \int_I dx\, \delta(f(x)) = \frac{1}{|f'(x_c)|}$$

$$\boxed{\int_A dx^N \delta(F[\Phi_c]) = \frac{1}{|\det \mathcal{J}_c|}}$$

what's this ?

Orbit Jacobian

$$J_{zz'}[\Phi] = \frac{\delta F_z[\Phi]}{\delta \Phi_{z'}}$$

matrix

The Law at neuron z

perturbed by wiggling neuron z'

perturb field configuration
everywhere, all at once

Φ_z

remember? stupid idea?

"euclidean"

$$d^2(\Phi_1, \Phi_2) = \begin{cases} (11.3\,\mathrm{mV})^2 \\ (\mathrm{calc./potasin})^2 \end{cases}$$

smart idea: matrix; determinant

can compute;

$$\left|\det J[\Phi_1]\right| = 21.3763419\ldots$$

for me. for you. everyone!

$$\left|\det J[\Phi_1]\right| = \prod_{\alpha=1}^{N} \Lambda_{1,\alpha}$$

eigenvalues

$$\Lambda_\alpha = e^{\lambda_\alpha}$$

$$\ln\left|\det J_1\right| = \sum_\alpha \lambda_{1,\alpha}$$

stability exponents

Compare Two States

$$\ln\left|\frac{\det J_2}{\det J_1}\right| = \sum_\alpha (\lambda_{2,\alpha} - \lambda_{1,\alpha})$$

"distance" in "Fourier" space

but, that's neuroscience!

$$p_1 = \frac{1}{Z} e^{-S[\Phi_1]}, \quad p_2 = \frac{1}{Z} e^{-S[\Phi_2]}, \quad p_3, \ldots$$

$$= \frac{1}{Z} \frac{1}{|\det J_1|}$$

$$\boxed{S[\Phi_j] = \ln \det J_j}$$

$$-\ln p_1 = \ln Z + S_1$$

information

$$\frac{1}{Z}\sum_c s_c\, e^{-s_c}$$

Kullback-Leibler divergence

$$D(\Phi_1 \| \Phi_2)$$

average over state Φ_1

$$= \ln\frac{Z_2}{Z_1} + \langle S_1 - S_2 \rangle_1$$

Example: Align 2 multivariate gaussians, given

$$D(\Phi_1 \| \Phi_2) = \frac{1}{2}\Big\{ (\Phi_2 - \Phi_1) J_2 (\Phi_2 - \Phi_1) + \underbrace{\ln\det\frac{J_2}{J_1}}_{\text{deterministic ``distance''}} + \underbrace{\mathrm{tr}\left(J_2^{-1} J_1 - 1\right)}_{\text{quadratic, variational error}} \Big\}$$

Ali Farshchian, Juan A. Gallego, Joseph P. Cohen, Yoshua Bengio, Lee E. Miller and Sara A. Solla
Adversarial domain adaptation for stable brain-machine interfaces, arXiv:1810.00045

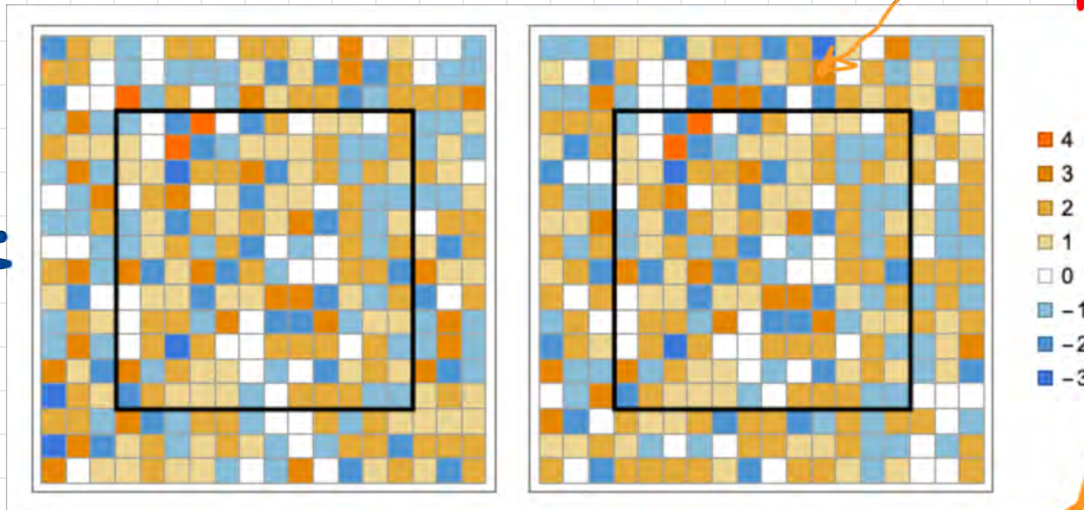

almost there!
eresfjord
24 july 2024

Q. How close are

neuron state, color coded

region $\mathbb{A}$

$\Phi_1 =$



$= \Phi_2$

not statistics

deterministic

A. $d(\Phi_1 \| \Phi_2) = \frac{1}{N_{\mathbb{A}}} \sum_{\alpha}^{\mathbb{A}} \left(\lambda_{2,\alpha} = \lambda_{1,\alpha}\right)$