ChaosBook.org chapter

counting

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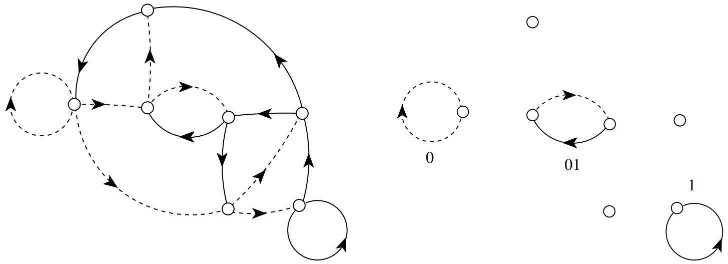
counting is a thrill

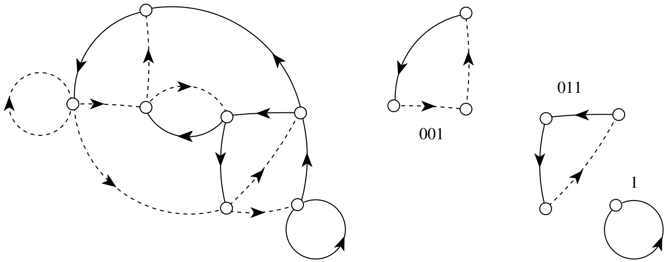
our hymn to counting is a symphony in two movements:

1. counting
   - counting equates multiplets of equivalent orbits.
2. counting

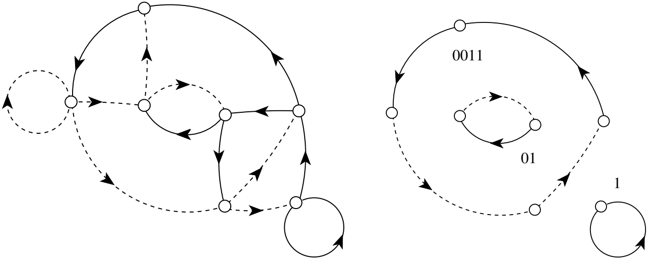
loop expansion of a graph



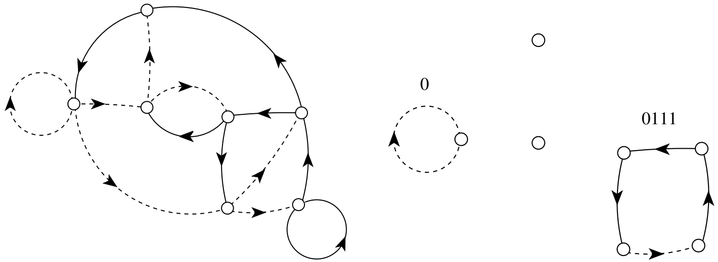
loop expansion of a graph



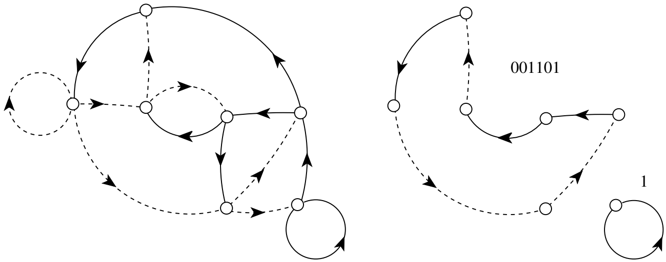
loop expansion of a graph



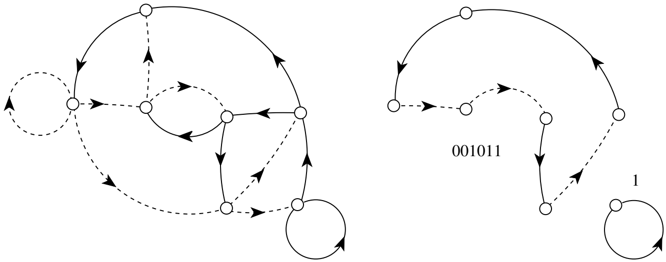
loop expansion of a graph



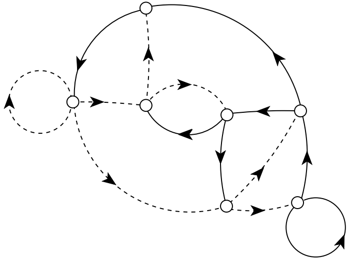
loop expansion of a graph

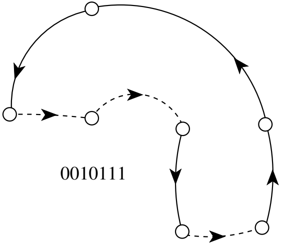


loop expansion of a graph

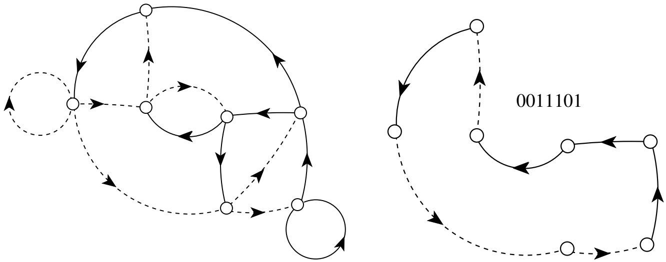


loop expansion of a graph





loop expansion of a graph



loop expansion of a graph

(a) The region labels in the nodes of transition graph can be omitted, as the links alone keep track of the symbolic dynamics.

(b)-(j) The fundamental cycles for the transition graph (a), i.e., the set of its non-self-intersecting loops. Each loop represents a local trace t_p.

loop expansion of a transition graph

Consider a state space covered by 7 neighborhoods, with the topological time evolution given by the above transition graph
The determinant $\det(1-zT)$ of the transition graph read off the graph as a polynomial in z; coefficients are products of non-intersecting loops (traces of powers of T) of the transition graph

$$
\begin{aligned}
\det(1-zT) = 1 - &(t_0+t_1)z - (t_{01}-t_0t_1)\,z^2 \\
&-(t_{001}+t_{011}-t_{01}t_0-t_{01}t_1)\,z^3 \\
&-(t_{0011}+t_{0111}-t_{001}t_1-t_{011}t_0-t_{011}t_1+t_{01}t_0t_1)\,z^4 \\
&-(t_{00111}-t_{0111}t_0-t_{0011}t_1+t_{011}t_0t_1)\,z^5 \\
&-(t_{001011}+t_{001101}-t_{0011}t_{01}-t_{001}t_{011})\,z^6 \\
&-(t_{0010111}+t_{0011101}-t_{001011}t_1-t_{001101}t_1 \\
&\quad -t_{00111}t_{01}+t_{0011}t_{01}t_1+t_{001}t_{011}t_1)\,z^7\,.
\end{aligned}
$$

fundamental cycles

Twelve cycles up to period 7 are fundamental cycles:
$\overline{0}, \overline{1}, \overline{01}, \overline{001}, \overline{011}, \overline{0011}, \overline{0111}, \overline{00111}$,
$\overline{001011}, \overline{001101}, \overline{0010111}, \overline{0011101}$,
out of the total of 41 prime cycles up to cycle period 7.

topological polynomial

The topological polynomial $t_p \to z^{n_p}$

$$1/\zeta_{\mathrm{top}}(z) = 1 - 2\,z - z^7$$

is interesting; the shadowing fails first at the cycle length $n = 7$, so the topological entropy is only a bit smaller than the binary $h = \ln 2$.

the main result of this chapter can be stated as follows: