10231: Nonlinear dynamical systems, fall 1998
Homework assignment 2

I.

Consider the system of differential equations

$$\dot{x} = y \tag{1}$$

$$\dot{y} = -\frac{2x}{x^2+a} - by + c \tag{2}$$

1. Find all fixed points for all values of the parameters a, b, and c, which are all assumed to be positive.
2. Find the stability of each fixed point.
3. Find the bifurcation point in the parameter space and the type of bifurcation, which occurs.
4. Show, for $b = 0$, that the motion is that of a 1-d particle moving in a potential. What is the potential.
5. Discuss the physical meaning of the parameters a, b, and c and of the bifurcation.
6. Show that there exists a homoclinic orbit as long as $ac^2 < 1$ and $b = 0$. Does it remain for $b > 0$? Sketch the invariant manifolds both for $b = 0$ and for $b > 0$.
7. Let $b = c = 0$. Find the frequency of the periodic motion for a given amplitude r to first order in the small quantity $\varepsilon = r^2/a$, i.e. of the form $\omega = A(1 + C\varepsilon)$, where A and C do not depend on r.

II.

Analyse the equation

$$\ddot{x} + x - (\varepsilon + x^2 - x^4)\dot{x} = 0 \tag{3}$$

1. Use the averaging theory (or equivalently two-timing) to argue that a bifurcation takes place when ε is varied. Determine the nature of the bifurcation and draw a bifurcation diagram.
2. Verify your results by direct numerical solution of the differential equation and plot the bifurcation diagram as well as characteristic phase plots.
3. Show that the system exhibits *hysteresis* when ε is varied. What is the largest ε for which hysteresis exists?

$\boxed{\textbf{Due at the lecture } \textit{\textbf{Friday, October 30.}}}$