10231: Nonlinear dynamical systems, fall 1998
Homework assignment 1

1. For each value of the parameter $r \in \mathbb{R}$ find the fixed points for the equation

   $$\dot{x} = (x-r)(x^2 - 2x + 2 - r)$$

   and classify them according to stability. Show that $r = 1$ and $r = 2$ are bifurcation points and determine the bifurcation types by drawing the bifurcation diagram. (*Hint:* Do not expand the parentheses!)

2. Sketch the phase portrait for the linear system

   $$\dot{x} = -x + 8y, \;\; \dot{y} = x + y,$$

   Include the nullclines (the curves $\dot{x} = 0$ and $\dot{y} = 0$, see example 6.1.1 p. 147) in your drawing.

3. Sketch the phase portrait for the linear system

   $$\dot{x} = x - y = f(x,y), \;\; \dot{y} = 2y - 2x = g(x,y).$$

   Note that the system is degenerate, and has infinitely many fixed points. (*Hint:* For two-dimensional systems it is sometimes useful to eliminate time from the equations and obtain an equation for the trajectories:

   $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\dot{y}}{\dot{x}} = \frac{g(x,y)}{f(x,y)}.$$

   Of course, this is possible only when $f(x,y) \neq 0$. For the present problem, the equation for the trajectories is easily solved.)

4. Sketch the phase portrait for the non-linear system

   $$\dot{x} = -x + y, \;\; \dot{y} = -x - y + 2xy.$$

   That is, find and classify the fixed points. Find and draw the nullclines. Sketch the phase portrait in neighborhoods of the fixed points and try to connect the orbits globally. Describe the basin of attraction of the stable fixed point in terms of invariant manifolds. Approximate the basin boundary near the saddle point. Do you think there are any closed orbits (periodic solutions)?

$\boxed{\textbf{Due at the lecture } \textbf{\textit{Friday, October 2.}}}$