spatiotemporal cats
or, try herding 7 cats

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2022-02-13 Josh & Sam Questions about how to best (and practically) evaluate cycle averaging formulas:

1. The numbers of terms in the expansion grows so quickly with respect to the minimal symbol length orbit excluded that we are not quite sure how and where to truncate the sum, even moderately sized collections of orbits.
2. Has anyone attempted to compute periodic orbits averages by numerically computing the zero and derivative of $F = \prod_p (1 - t_p)$ directly?

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1. Nobody so far has had enough understanding of Navier-Stokes periodic orbits to evaluate truncation errors. For low-dimensional systems:
   (a) If grammar is known, exponentially decreasing errors kick in only after 'fundamental' cycles are accounted for, read the end of ChaosBook sect. 18.3 *Determinant of a graph*
   (b) If symbolic dynamics is not understood, ChaosBook sect. 23.7 *Stability ordering of cycle expansions*
2. None has attempted it - an idea worth exploring.
   (a) Watch out for ChaosBook sect. 22.4 *False zeros*: the unexpanded product $\prod_p(1 - t_p)$ is only a shorthand, just like for the original Riemann zeta function.
   (b) If you the terms as a (pseudo)cycle expansion, numerically "computing the zero and derivative" seems to be what we already do?
3. But your question does lead to something that Matt Gudorf never explored in his thesis: Perhaps the most important insight of the spatiotemporal reformulation of 'chaos' is that the weight of periodic orbits (N-torus, if theory has N continuous symmetries) is given by its Hill determinant, see LC21 sect 8.2 *Periodic orbit theory for the retarded*.
   (a) Can you think of new/better ways to evaluate $\mathrm{Det}\,\mathcal{J}$? Orbit Jacobian matrix $\mathcal{J}$ is big, but very sparse, and $\mathrm{Det}\,\mathcal{J}$ has a nice geometrical interpretation as a LC21 fundamental parallelepiped? The edges of the parallelepiped are the columns of the Orbit Jacobian matrix, which are sparse, so maybe it is computable?
   (b) In the continuum limit (more appropriate to Navier-Stokes?), maybe the best was is to follow LC21 Hill and Poincaré, and truncate Fourier series?
   (c) For viscous flows, like Navier-Stokes, the infinity of transient, strongly dissipative modes immediately damp put, so the Hill

determinant should only have the dimension of the *inertial manifold*. Does it?

2022-02-19 Predrag JAX is said to make evaluation of Jacobians trivial.