Bohr,~N.
Feynman,~R.P.
Rugh,~H.H.
Viswanath
Lautrup,~B.

# Appendix Q

# What reviewers say

## Q.1 Niels Bohr

"The most important work since that Schrödinger killed the cat."

## Q.2 Richard P. Feynman

"Great doorstop!"

## Q.3 Hans Henrik Rugh

Old and confused.

## Q.4 Divakar Viswanath

I have read the first six chapters. The introduction chapter is fabulous. The computation of the escape rate for the pin-ball game in the first chapter is absolutely riveting. There can't be a better way to begin. I find the book to be very stimulating.

## Q.5 Benny Lautrup

I am now reading your book as meticulously. I have lots of little comments, and one big one, which I can tell you immediately. I think that your opening chapter with its many literary references, jokes, and finesses, loses out in the end, because the main message gets obscured. Don't make section

Gatto
Nero!professor
Nero, G.

quotes unless they are really pertinent (although I love the first one about standing behind giants, but why cite Kierkegaard in Danish (or at all for that matter)?

## Q.6 Professor Gatto Nero

This book, which I have received unsolicited from the Szczsyrk Oblast Agricultural and Mazuth Office Press appears to be a collage of LaTeX clips from random papers authored by the motley collection of co-authors whose number one usually associates with an experimental high energy *Phys. Rev. Letter*, rather than a book that aspires to be the Landau-Lifshitz of chaos.

Entire rain forests went down so this not inconsiderable tome can be printed and bound. Why these ravings were not left on the Web where they more properly belong is not explained anywhere in the text. Indeed, what is worrying about this book is that in an opinion poll about whether the book should be printed on paper or left on the Web, almost no graduate students expressed opinion, one way or the other. Perhaps this means that there is no "market" out there, and publishing just another book is a fair way to give up on more ambitous schemes.

As far as to the value of webbooks, let us turn to Eric Heller's thoughts on the matter: "What sort of snootery would keep people away from an authoritative text on the web? The problem our colleagues have identified is authoritative. In other words, the chapters are not 'stamped' as 'published'. Why not do that as sections are really finished? On can still revise later or even frequently. The software paradigm is a good one. Written by many people, the webbook is updated, bugs are fixed, there are beta versions, etc. As long as somenone is willing to oversee the project, I think the webbook is a great experiment. I use this book a lot, and would like to see 'live' books flourish."

I find the idea of a constantly updated webbook with hyperlinks and so on magnificent but still too modern. Readers and authors prefer a physical book. I have great difficulties lying in the bed reading on a 17" screen on the belly, and its not fun to print $10^3$ pages when they are updated next month.

One issue one should consider is whether the book could be split into two volumes, one classical and one quantum. However, the quantum chaos part is just an application of the ideas learned from the classical periodic orbit theory. It is not something that stands alone. One should limit the quantum part to zeta functions/periodic orbits, without dealing with the statistical properties of spectra, etc. . A single book 80% classical + 20% quantum seems appropriate, without splitting, since one cannot fill an entire book with interesting quantum stuff.

Indeed, I wonder how seriously to take the notion that the book will be used as a graduate student text. Certainly the authors may use it as such but I wonder how many others would. As a guide to the level of the discussion, saying that it is for advanced graduate students is fine. I just see it being more of a reference text than an actual course text. And, as the things change rapidly and the whole concept will be ultimatelly outdated within a few years.

There are lot's of Lifshitzs listed here, but no L.D. Landau, and it shows. Reading the book I am reminded of the 60's, when I went to a supermarket with my newly formed collective. He did not like cronflakes, she did not like cheese, the third one hated broccoli, and so on - by the time we reached the checkout, we had only toilet paper and the dish detergent in the carriage. So it seems to have been with these "co-authors": one does not like Henkel functions, the other one does not understand the Green-Kubo formula, while the third one hates fractals. By the time they agreed on the overlapping set of what they all understood, they were left with 700 pages of formalism and no applications.

The number of this book's contributors leaving academia is increasing day by day. Authors desparately need all credit they can get, and in Sweden they fire people who contribute to such books. Nevertheless, as the letter appended to the end of this review attests, the true autorship of this book is in doubt. The authors should paid the due respect to the unprovably mafioso italian christian democrat Professore Dottore Gatto Nero, equipped him with a homepage, CV and a publication list, and foisted off all the ill repute of this book on him. While he would be pictured with Pope on the homepage, the authors could labor annonimously under this Spirito Santo, for the love of science, their reputations unsmirched? The ultimate revenge on all the annonymous referees that have done the authors wrong over the years - the annonymous authors!

And so the whole book starts making sense. If it is an antiBourbaki, as one has in the antimatter world, then why not do the real thing? A Landau-Lifshitz for nonlinear Science, written as it should be done. The nonScience book to end all nonScience books.

## Q.7 His Holiness the Pope Gregorius III

This Gatto Nero - whence does he appear again? No one wants him and it is not even funny. But never mind: he started up with the webbook again, so let me spell out the sancrosanct dogma again: to have a good webbook you need

I) at some time $t = t_0$ a well written and complete and coherent manuscript including all references, exercises etc etc - in other words a book

II) you then have to implement web-specific features like hyper-links, video clips and so on

III) Then update the text for $t > t_0$ on a regular basis (why else would you want to have a time-dependent, non-static web-book). You include new material, take obsolete stuff out and you do all this in a way that does not disturb the rest of the text, which means updating all the text in the book from time to time.

In order to do this, you need a well disciplined team of authors and you need to be willing to do much more work than for a book only. Both these conditions are not fulfilled in the case of the coauthors of the book in question. They are barely able to reach point I) on the list above, I doubt that there is anyone who is willing to put the extra effort in to make this an exciting, colourfull, sparkling web-site with all the blinking web-features (point II) and there might be no one around in 5 years time to update the webbook at all. So if all they produce is a static Web-book, why than not publishing it straight away instead of letting it deteriorate at the web again. Sorry gentlemen, but that's all what you are in for.

### Q.7.1 Biographical note

His Holiness the Pope Gregorius III (731-741) was called by the people "The friend of the poor and miserable". His Holiness passed away on the 27th of November of the year 741 after being at the head of Christ's Church for ten years, nine months and twelve days, in difficult times and during complex international changes. The church celebrates in his honor on the 28th of November. It is written about him in the books of the Roman Popes: "He would add to wisdom the knowledge of the holy books and he was an outspoken preacher and an eloquent speaker".

## Q.8 Coauthors' combined comments

But let us turn to the text itself, chapter by chapter. As the strong sides of this text are exceedingly few, I will concentrate here on the severe shortcomings that jump at even most casual reader.

First, some overall problems that plague the entire text:

- Figures are awfull. While that might be a technical nightmare, the book needs need more illustrations and cleaner graphics throughout.
- Not enough applications.
- The chapters do not fit together, even the overall logic is not clear sometimes.
- Things are not well defined, too vague etc
- Important parts are missing

- There are too many different styles in this book
- resolve conflict with pdfmark package for mkpdfRaw which prevents generating hypertext version of coauthor's copy
- bibliography references format in the text now wrong; refers to [27] rather than the correct chapter number format [11.27] for ref 27 in 11th chapter.
- US letter size mkpdfRaw - decrease pagesize so all margin pointers are visible
- US letter size - public version: shift top margin down
- make a table of tables?
- insert someplace: Hausdorff-Baker expansion, ...

### Contributors

decrease the interline spacing in the itemize lists

### Preface

## 1 Overture

I have to say that the melancholy authors sound jaded and too impatient to explain carefully. Is this the new cool? No cutting is needed, just a bit more care in explaining. I reread chapt. 1 again and again until my strength gave out, and do not adapt to it. The style is agreeable but I think what bothers me is that it is trying to be introductory and goes into technical talk. That is fine but when I get my mind set for technical talk I want to get the real thing and then it turns out that the effort was wasted because it does not go all the way and simply leaves me hanging saying we'll do this later. This kind of teasing is not for me. Either do it or don't do it is what I say and I am for less technical stuff in the intro and then getting down to it for real when it happens.

### 1.1 Why this book?

### 1.2 Chaos ahead

I cannot find any place where "chaos" is clearly defined in this book.

As far as I know, no smooth analytic flow with compact phase space is ergodic (meaning only unstable periodic orbits, no marginal, elliptic or attracting cycles), and my belief is that there might eventually emerge a general proof that smoothness and compactness requires some islands of stability.

I know of 3 candidates on the market.

1) for many years it was believed that 2-dimensional $(xy)^2$ potential is chaotic. This was killed by P. Dahlqvist and G. Russberg, *Existence of stable orbits in the $x^2y^2$ potential*, *Phys. Rev. Lett.* **65**, 2837 (1990).

2) Gutzwiller still believes that for sufficiently large anisotropy anisotropic Kepler problem is chaotic, and has done extensive numerics to check that. I (and, for example, Devaney) believe that the islands are most likely so small that one cannot find them without detailed topological intuition of where they woould pop up (that's what was needed to kill the conjecture 1).

3) A. Knauff has proven that certain 1-$d$ array of scatterers is ergodic - however, this does not qualify as the potential is piecewise analytic, not one analytic function.

Arguments that ergodic flows might nevertheless exist are based on studies of the Henon map, idea being that a Poincare section of a stertch&fold flow might be reducible to map of Henon type; there we believe that in positive measure in the parameter space it is possible to recursively change the parameters in such a way that the period of the shortest stable cycle can be bounded to be arbitrarily long. For $(a, b)$ values close to the $(2, 0)$ 1-dimensional Ulam quadratic map this has been proven by Bennedicks and Carlson. But this requires fiddling with parameters - when one is given a potential like $(xy)^2$, there is not much to fiddle with.

It is very easy to construct nice chaotic flows if one is willing to accept repellers; 3-disk repeller that the book uses as the teaching example is a nice model of this type.

## 1.3 A game of pinball

## 1.4 Periodic orbit theory

## 1.5 Evolution operators

## 1.6 From chaos to statistical mechanics

## 1.7 Semiclassical quantization

"Dear Professore Nero,
I am from Australia, and I am planning to do my project on quantum

chaos, but am having trouble coming up with a suitable definition, let alone a specific area to study. I have downloaded and read several chapters of your book, as well as reading parts of Gutzwiller's, but have had trouble in obtaining a useful overall picture of the subject. Can anyone tell me what on earth is quantum chaos? "

Dear Michael,

An answer: There are several definitions. The only sensible one is Berry's "quantum chaology", quantum mechanics of systems that are classically chaotic - in this book the connection is provided by Gutzwiller trace formula and its generalizations. One can try to define 'quantum chaos' by searching for positive Lyapunovs and positive entropy in the space of time-dependent wave functions, but that tends to lead to a muddle, as people can then prove that 'quantum chaos' does not exist, and become very excitable (late Joe Ford). Third approach is to posit that spectral correlations are random (modulo symmetries such as the time reversal) - started with Wigner - and describre systems with such quantum spectra as "chaotic". This conjecture has not - so far - in any way been connected to classical chaotic dynamics, so I find it not too useful. People studying conductance fluctuations in mesoscopic systems find this useful, though.

All that is accomplished here is a rather fancy set of techniques for systematic semiclassical quantization, just a version of WKB or EBK quantization for chaotic systems. In no way replacement for Schrëdinger equation, but only a chapter 20 or so in a good graduate quantum mechanics textbook of the future.

## 1.8 Guide to literature

I was struck by the clarity of some of authors' transparencies shown in conference talks, especially with respect to trace functions and evolution operators. Although some of these diagrams have in part surfaced within the text, the text tends to be more formal in tone. The authors should consider placing more explanatory "figures" along the lines of their conference transparencies. One might set some of these off as outlined boxes on their own page, to isolate their informality from the text. Placing such figures at reasonable places in the text would be an educational asset - it is what people would likely get attending the author's courses in person, but its absence makes the material less penetrable to the isolated reader.

## Guide to exercises

## Resumé

## Exercises

# 2 Trajectories

NW: The analogy to celestial firmament seems a little forced to me. Also, the analogy is tenuous. In order to predict the future positions of planets relative to the background stars, it is necessary to know their current momenta as well as positions. So the firmament is not really the phase space — the phase space is something like the tangent bundle (or cotangent bundle for a Hamiltonian formalism). This is apt to confuse since when you deal with dynamical systems, it is usual that the velocities are phase space variables while this analogy implies that they are not. (And this doesn't even mention that the planets are evolving in 3 dimensions and the firmament is a two dimensional projection.)

## 2.1 Flows

PC: the notion of *attractor* is not defined anywhere! Fix.

## 2.2 Computing trajectories

Sect. 2.3 needs to be written.

## 2.3 Maps

(Gabor Vattay 11 Mar 1999): In order to motivate the qualitative dynamics chapter I need specific differential equations. I also need them to motivate diff. eqations —¿ maps change. Rossler is good, but nobody in my class understood what is on the pictures with dogs. They are not scientists, they should be able to figure it out from the book.

## 2.4 Infinite-dimensional flows

Insert figure of a spatiotemporal solution

## 2.5 Billiards

Write a clear and implementable billiard dynamics algorithm (no arcsine functions and such - at most a square root)

### Resumé

Resumé still needs to be written.

### Exercises

# 3 Local stability

PC: Very serious weaknesses in this chapter, stability stuff has not been properly written up.

Goals of this chapter (Gabor Vattay 11 Mar 1999):

Draw an attractor, show chaotic systems and to compute stability, show Poincare map —- can preceed symbols. This is extremally helpful for students, who do not take a class on pure chaos beforehand. (60 % of my students are such students.) If we could start with such chapter, it would make possible to use our course as a self-contained chaos course.

## 3.1 Flows transport neighborhoods

Define the Lyapunov exponent already here, by transferring a non-cycles part of sect. **??** to this chapter.

## 3.2 Linear stability of maps

## 3.3 Linear systems

This section lacks all the essential material written up in any textbook on nonlinear dynamics. This must still be written up.

## 3.4 Understanding flows

Looks indigestible for a student in the present form.

## 3.5 Stable/unstable manifolds

Christiansen believes that for the Rössler system he got a result accurate to machine precision and concluded that the eigenvalue was at least as small as that. H.H. Rugh and Christiansen chose the Lorenz attractor as demonstration in their Gram-Schmidt/Lyapunov paper, not the Rössler. In Lorenz the eigenvalue is also quite small - the exponent is of order of -14, but there is a nice check since the sum of the exponents is given by the equations. Even though the Rössler is not quite so nice, one can make a reasonable estimate and find that the eigenvalue is indeed on the order $10^{-35}$ per revolution or whatever.

If Christiansen finds the time, he might stuff the Rëssler equation into the Gram-Schmidt program.

## 3.6 Stability of periodic orbits

This section lacks all the essential material written up in any textbook on nonlinear dynamics. This must still be written up.

The topic of finding the Poincaré monodromy matrix for a Hamiltonian flow from the full Jacobian is discussed in appendix C. Whelan prefers Christiansen's exposition in chapter 8. There is also the issue of doing that for an arbitrary (non-Hamiltonian flow) which Christiansen discusses.

### 3.7 Billiards

One of the worst sections of the whole book: The Bunimovich-Sinai curvatures are not well written up, billiard stabilities are inadequately discussed and immensely important. Gaspard does a nice job in his book, worth having a glance at.

#### Resumé

Resumé still needs to be written.

Bibliography is missing.

#### Exercises

## 4 Transporting densities

Maybe I missed some conceptual overall plan, but I do understand the philosophy behind this chapter. Are conservation equations used in following chapters? In deriving the exponential form for $U_t$ one should mention the Stone theorem, as the average reader is likely to know a little functional analysis: and the emphasis should be on infinitesimal generators. If functional analysis is to be avoided why mention is made to pointwise convergence of $U_t\rho$ ? The remark on Koopman operator probably would be worth more space: it is true that zeta functions require restricting $L_2$: but this is one of the crucial issue of all the story, and the physical import (...) is that correlation functions may decay how they like in a too big space: moreover I wonder whether the introduction of Koopman operator might be the right point to briefly mention how spectral properties of $U_t$ are related to ergodic properties.

### 4.1 Measures

### 4.2 Density evolution

### 4.3 Invariant measures

## 4.4 Evolution operators

### Resumé

### Exercises

# 5 Averaging

## 5.1 Dynamical averaging

Redraw figures 5.1, 5.2.

## 5.2 Evolution operators

### Resumé

Move the "pressure" remark to a more appropriate chapter.

### Exercises

# 6 Trace formulas

(Gabor Vattay 11 Mar 1999): in my course I skip the trace formulas. I don't think the trace formulas are important. If you teach somebody who neither saw a trace nor a determinant, can immediately learn the correct thing. We dont have to fight with shadows of our research community.

## 6.1 Trace of an evolution operator

## 6.2 An asymptotic trace formula

### Resumé

### Exercises

# 7 Qualitative dynamics

This material is really turbulent, the main problem is that this is research based and not undergraduate classroom material. Everything needed is in this chapter, it just looks like a salad. The chapter should be improved. Either one should be easy going and say less and just to show few examples, or one should define concepts more correctly like in the appendix B. So, I skipped pruning fronts and most of the pruning, since I am not an expert in this. The problem with the pruning part is that there is no way to figure out what is the most complicated pruning and if say 00 is excluded then it will be excluded in all circumstances, so what?

### 7.1 Temporal ordering: itineraries

### 7.2 3-disk symbolic dynamics

### 7.3 Spatial ordering of "stretch & fold" flows

### 7.4 Unimodal map symbolic dynamics

### 7.5 Spatial ordering: symbol plane

### 7.6 Pruning

### 7.7 Topological dynamics

The general idea of template analysis is appealing, but it makes no appearance in this book. I feel that some template analysis should be included, but would fear even starting - because I see no quick and simple exposition, no way to present it in 10-20 pages and still enable a student to use it. Has somebody written a brief but not superficial text that lends itself to a few lectures and a problem set in the style of this book?

#### Resumé

Edit references.

#### Exercises

## chapter 17 Fixed points, and how to get them

Christiansen plans to finish off a beautiful chapter during the month of August, 2000. But don't hold your breath - he has to prepare opening session talks for the European Geophysical Society's yearly symposium, and such.

## 8.1 One-dimensional mappings

## 8.2 $d$-dimensional mappings

## sect. 17.5 Flows

I am interested in applying this material to determine numerically the unstable periodic orbits of some nonlinear ordinary differential equations. In particular, I am working on (1) periodically driven Van der Pol equation, (2) a system of 3 coupled ODE's describing the nonlinear coupling of three waves as studied by Wersinger and Ott.

I found the methods desribed here very efficient for the case of maps and wondering if you know of any similar algorithms or papers that I can use to find the unstable periodic orbits of my differential equations. So far I were unable to find them numerically. When I integrate my equations backward in time the solutions tend to converge to a different unstable periodic orbit from that I am looking for. For instance, instead of period two orbit it converges to a period one orbit.

Most chaos books only discuss the above problem in the context of maps, without paying attention to differential equations. It would be nice if your could elaborate your chapter further to include an algorithm for the case of ODEs such as the classical driven Van der Pol equation. This will be of great help to the community since often we have to handle ODEs rather than maps.

## sect. 17.5.2 Newton's method with optimal surface of section

Niall Whelan (23 Apr 2002):

I looked at Carls' notes.... they are not the absolutely easiest to read. I think we have differing definitions of D. I thought it was the dimensionality of the configuration space so 2D was the dimensionality of the phase space and 2D-2 was the dimensionality of the Poincare' section (for a Hamiltonian system, the Poincare' section reduces the dimensionality by two.) I believe for Carl (and you?) D is the number of crossings of the Poincare surface, which is obviously a different thing.

What I was talking about was the equivalent of going from Carl's 3x3 matrix J to his 2x2 matrix $\tilde{J}$, but where you can reduce the dimensionality by 2 as opposed to 1 (by using the time flow and also using the Hamiltonian conservation.) I wasn't referring to multiple mappings which seems to be his point.

I could be more explicit in writing out what I mean if you care, but it now seems that we were talking about different things. (As in other topics involving symplecticity (an 80's Police album? something I learned from Stephen.)

## 8.4 Sundry methods for cycle extraction

Discuss extraction of long time series.

The "extras" homepage contains a contribution from F.K. Diakonos, D. Pingel and P. Schmelcher: *Systematic Detection of Unstable Periodic Orbits in Discrete Chaotic Dynamical Systems* (4 July 2000). The method to detect orbits is of general character and works very well, at least for the authors. They have combined it with the existing Newton-Raphson approach and thus it becomes also more efficient and works perfectly. It can also be applied to higher dimensional systems and goes to much higher periods than, for example, Newton-Raphson on its own. Shouldn't this be included here?

Thomas Schreiber (Aug 9, 2000) and Predrag Cvitanović bemoan absence of a good review on extraction of cycles from DATA. Thomas is sceptical about the Schiff/So approach; if you define a periodic orbit to be something that your software prints, then you can do nice things. Otherwise...

There is some work by Schmelcher and Diakonos which is in the class of Biham&Wenzel old paper on Hénon cycles. They claim it can be done from a time series but seem to have not gone beyond numerical Ikeda data. The review on NMR laser chaos by Brun, Badii etc (Rev.Mod.Phys.) is somewhat special since the system is rather structured (almost 1D Poincare section).

As far as Thomas knows, there is no way around modeling the full system first and then find UPO's of the model. Kantz and Schreiber do that for locally constant models in the routine in the TISEAN package, others have done it with RBF, neural net and other models.

## 8.5 Periodic orbits as extremal orbits

The statement that cycles are minima of the length function is wrong. They may be maxima and saddles as well, think about the stadium for two seconds. It think they are always minima in dispersive billiards, though.

**8.6 Stability of cycles for maps** Roberto: I am trying to collect observations/references on various points (for instance i cannot find where it is expained that stability is the same along all cycle points ..).

—-

Write up a clear argument that stability eigenvalues are invariant for maps and flows in d-dimensions.

### Resumé

### Exercises

# 9 Counting

## 9.1 Counting itineraries

## 9.2 Topological trace formula

## 9.3 Determinant of a graph

## 9.4 Topological zeta function

## 9.5 Counting cycles

## 9.6 Infinite partitions

### Resumé

### Exercises

# 10 Spectral determinants

The idea is to introduce concepts thorugh a series of toy examples. The logic should be something like this:

1. 1-$d$ linear fixed point
   (a) unstable: $k = 0$ piecewise constant
   (b) stable: $k = 0$: $\delta(x)$ singular
2. 1-$d$ piecewise linear map
3. 1-$d$ nonlinear fixed point
   (a) unstable: $k = 1, 2, \ldots$ analytic $\rho$
   (b) stable: $k = 1, 2, \ldots$: $\delta(x)^k$ singular
4. 1-$d$ piecewise analytic map
5. 2-$d$ linear hyperbolic fixed point
   $\rho =$ smooth$\times$singular
   future$\times$past $=$ smooth $\rho$

Niall D. Whelan (11 Oct 2002): - include a flowchart showing the connections among the propogator L, the trace formula, and the various zeta functions. Having something visual as an aid would help to keep track of the thread of the development. I could sketch out something rough in

### 10.1 Spectral determinants for maps

### 10.2 Spectral determinant for flows

### 10.3 Dynamical zeta functions

### 10.4 The simplest of spectral determinants: A singel fixed point

### 10.5 False zeros

### 10.6 All too many eigenvalues?

### 10.7 More examples of spectral determinants

### 10.8 Equilibrium points

### Resumé

### Exercises

## 11 Cycle expansions

### 11.1 Pseudocycles and shadowing

### 11.2 Cycle formulas for dynamical averages

### 11.3 Cycle expansions for finite alphabets

Someplace write up and describe Liouville spectrum for classical 3-disk systems.

The in the quantum part compare the classical and the quantum spectrum.

### 11.4 Perturbation expansions in terms of cycles

### 11.5 Stability ordering of cycle expansions

### 11.6 Infinite alphabets

### 11.7 Equilibrium points

### 11.8 Dirichlet series

### Resumé

### Exercises

## 12 Getting used to cycles

This chapter, which in the preceeding edition of the webbook purported to make the formulaic incantations of the preceeding eleven chapters understandable and accessible to mere humans, has been edited with sledge hammer. Vattay got that Copenhagen feel-good feeling bashing at it, probably just because he works at night powered by Tuborg beer, not Carlsberg. Then he went to sleep, and the chapter consists of shards that survived to see the "first serious DasBuch version of the 21 century".

Problems with Vattay's rewrite: the topics introduced in the chapter have not been indexed, no figures illustrating them have been prepared, cross-referencing with the rest of the text (in the preceeding version of the chapter) has been destroyed, the references have not been updated, cross-checked and cleaned up.

### 12.1 Escape rates

I am still confused about something. If I take the standard map and jam a cubic function into its inerds, – say at $x = 0$ – I will get an isolated (nonmixing) region surrounding $x = 0$. If I fiddle the parameters of the cubic thingee, I can vary the leakage from the isolated region to be just about whatever I want. To me, for a tiny rate of leakage up to a large rate, the topology – and hence the symbol sequences if I could partition the problem – ought to be essentially the same. And of course I can set the eigenvalues at $x = 0$ to be whatever I choose, independent of the rest of the cubic function. So I would think that periodic orbit theory would say that the low-leakage and the high-leakage problems have similar measurables, yet the mixing changes fairly dramatically with the rate of leakage. So what am I missing?

## 12.2 Trace formulas *vs.* level sums

## 12.3 Eigenstates

Carl Dettmann is disappointed that Vattay has removed Rondoni's exposition of orbital measures. Orbital measures are an important contribution to the subject, in that they

(a) Provide an alternative way of looking at periodic orbit formulas

(b) Are simpler to understand, and so benefit the reader

(c) Are simpler to calculate, for example applications in statistical mechanics

(d) Appear in quite a lot of literature, which would otherwise be quite confusing to the readership of the book.

Their application has so been almost all restricted to trace formulas, which are very much less convergent than zeta functions or spectral determinants in the best case (uniform hyperbolicity, finite topology), and so treatments of periodic orbit theory using only trace formula based orbital measures are very incomplete, however

(i) There is ample discussion of these other methods in the book, so the reader will never get the impression that orbital measures are all there is.

(ii) For weaker chaotic properties (the extreme example would be the randomly oriented Ehrenfest model, which is an infinite polygonal billiard with diffusive properties intimately related to the periodic orbits, see `nlin.CD/0001062`), it is not clear how to extract the most information from the periodic orbits, and trace fromulas may do as well as other methods.

(iii) Zeta function and spectral determinant expressions for averages are linear combinations of the observable computed on the periodic orbits, so they also correspond to orbital measures, albeit more complicated than those from trace formulas; thus the concept of "orbital measure" is not restricted to trace formulas and their slow convergence.

Dettmann hopes orbital measures will live again in the pages of the book, for they are very much alive elsewhere.

### 12.4 Flow conservation sum rules

### 12.5 Why not just run it on a computer?

### 12.6 Ma-the-matical caveats

### 12.7 Cycles as the skeleton of chaos

### Resumé

### Exercises

## 13 Applications

Problems with Vattay's rewrite: the topics introduced in the chapter have not been indexed, no figures illustrating them have been prepared, cross-referencing with the rest of the text (in the preceeding version of the chapter) has been destroyed, the references file (now renamed to refsAppApplic.tex) has simply been forgotten - not been updated, cross-checked and cleaned up.

The advanced parts of this chapter were moved into appendix on "Applications", the definition of Lyapunov exponents and correlation functions into getused.tex chapter.

**13.1 Lyapunov exponents** Moved into getused.tex chapter.

**13.2 Evolution operator for Lyapunov exponents** This section, based on (*inter alia*) a paper by Vattay *et al* has been moved into appendix on "Applications".

**13.3 Correlation functions** Moved into getused.tex chapter.

This needs serious improvement.

Vattay removed the Green-Kubo formula, as at this point it preceeds the chapter on the diffusion constants, and suggested that Artuso incorporates it somewhere in the diffusion constant chapter.

**13.4 Advection of vector fields by chaotic flows** This section, based on (*inter alia*) a paper by Vattay *et al* has been moved into appendix on "Applications".

**Resumé** The "Resumé" was a remark on role of noise in dynamics, a topic discussed nowhere in this book, and certainly not in this chapter. Moved into a remark in getused.tex chapter.

### Exercises

# 14 Thermodynamic formalism

Problems with Vattay's rewrite: the topics introduced in the chapter have not been indexed, no figures illustrating them have been prepared, cross-referencing with the rest of the text (in the preceeding version of the chapter) has been destroyed. Need to extract refsThermodyn.tex from refsAppApplic, the references have not been updated, cross-checked and cleaned up.

Career update: Starting 8 of July Vattay runs his Communication Networks Laboratory founded by Ericsson on the Eotvos University. The topics are interesting: random graphs, stochastic maps and chaos on internet in general. But then one has to deal with real word engineering as well.

## 14.1 Rényi entropies

Vattay has elimanted the relation

$$K_1 = \frac{\langle -n\gamma + \lambda \rangle}{\langle n \rangle} = \lambda_0 - \gamma \tag{Q.1}$$

Restore, should also probably be generalized to continuous times.

## 14.2 Fractal dimensions

### Resumé

### Exercises

Vattay could not find a solution file to exerThermo. Somebody who wrote the original exercises might have a soluton to the first 3 problems and can add it to soluThermo.tex. Sure.

# 15 Intermittency

This chapter needs more figures.

## 15.1 Escape, averages and periodic orbits

### 15.2 Knowing your enemy

### 15.3 Defeating your enemy - Resummation of intermittency

### 15.4 Marginal stability and anomalous diffusion

### 15.5 Probabilistic methods and approximate zeta functions

#### Resumé

A more exhaustive guide to literature would be welcome?

#### Exercises

## 16 Discrete symmetries

### 16.1 Preview

### 16.2 Discrete symmetries

### 16.3 Dynamics in the fundamental domain

### 16.4 Factorizations of dynamical zeta functions

### 16.5 $C_2$ factorizations

### 16.6 $C_{3v}$ factorization: 3-disk game of pinball

### 16.7 Summary

#### Resumé

#### Exercises

## 17 Deterministic diffusion

Perhaps the Green-Kubo formula can be rescued and (re-)incorporated somewhere in this chapter.

### 17.1 Diffusion in periodic arrays

### 17.2 Diffusion induced by chains of 1-$d$ maps

### 17.3 Lorentz gas

### Resumé

### Exercises

## 18 Semiclassical evolution

Most of the figures need redoing. Tanner has changed the notation in f_hamilton_wave.eps from S to R, but also that picture needs some brushing up.

### 18.1 Quantum mechanics: a brief review

Tanner has removed the average density of states section, as his intention is to include it in the semiclassical quantization chapter, where it fits better.

### 18.2 Semiclassical evolution

Tanner has moved Madelung fluid/ hydrodynamical interpretation into a remark.

### 18.3 Semiclassical propagator

Tanner has the semiclassical propagator derived directly from short time behavior (read this section carefully); he choose to do it this way, because introducing the momentum picture is too ad hoc; if we want to start from momentum space, then we need to introduce wave function representations (configuration/momentum), Fourier transfrom and also stationary phase approximation here.

### 18.4 Semiclassical Green's function

Tanner has taken out the Gaussian approximation; not needed here, only the imaginary part is needed here.

The references are still missing.

### Resumé

### Exercises

Many solutions, to be included in soluSemicl.tex, are still missing.

# 19 Semiclassical quantization

## 19.1 Trace formula

## 19.2 Semiclassical spectral determinant

## 19.3 One-dimensional systems

## 19.4 Two-dimensional systems

### Resumé

### Exercises

# 20 Semiclassical chaotic scattering

The goals of this chapter: The derivation of Gutzwiller trace formula (as well as the classical trace formulas) given above was heuristic, and it is not clear when it is applicable, and whether it is correct. For example, in its derivation we assumed that the system is bound and the spectrum is discrete - nevertheless, our main application is to spectrum of helium, much of it above the ionization treshold. Wirzba's treatment answers several of these questions; the derivation is rigorous for a class of scattering problems. It also introduces new physical ideas in this set of applications, in particular the phase shifts. It derives cycle expansions from the exact cumulant expansions for Fredholm determinants. Together with the appendix, it connects this subject to the well developed theory of spectra of linear operators on Hilbert spaces. Without Wirzba's treatment we would have not understood where Gutzwiller-Voros zeta fails, trusted our creeping formulas, been able to go onto other problems, such as elastodynamic resonances.

In May 2000 Gregor and Andreas have agreed on how to reorder Andreas' original version of this chapter. Gregor thinks that examples about the 1-disk and 2-disk and/or 3-disk system should be given

But the chapter took much more radical turn.

One immediately notices that the author has shortened the chapter tremendously, basically by removing any reference to any Bessel and Hankel function and to any of Wirzba's published or unpublished papers.

The reason is clear - the author got rather depressed writing the chapter, feeling pressure to write once more the old thesis or even to improve on it (i.e., make it more pedagogical). After really getting in a very depressive mode, the author decided to NEVER EVER write about this stuff again. This is done in Wirzba's Physics Reports, it is the final word about the subject and it may rest there in peace.

So the author concentrated this chapter on the only thing a student should know about this subject, namely the Krein-Friedel-Lloyd stuff. Everything left out does not really belong in a book meant for graduate students.

As the new scattering chapter became very short and, in fact, is only about the Krein-Friedel stuff, one might make it to a new appendix. Indeed it is really written as an appendix or after-thought to chapters 18 and 19. Then, one might drop the exercise file which anyhow contains just one exercise. Furthermore, Wirzba's appendix about trace-class etc. is really not needed as far as this revised chapter is concerend. It could be dropped as well, very it not referred to throughout the book.

small typos etc., cannot be exclude since the author got feed up with the whole stuff. Anyway, the chapter is short and general enough that any pfhysicist off the street should find it easy to repair in a way that suits it suits her.

## 20.1 Quantum mechanical scattering matrix

## 20.2 $N$-scatterer spectral determinant

## 20.3 Krein-Friedel-Lloyd formula

## 20.4 Semiclassical reduction for one-disk scattering

## 20.5 From quantum cycle to semiclassical cycle

## 20.6 3-disk system

## 20.7 Heisenberg uncertainty

### Resumé

### Exercises

# 21 Helium atom

Tanner's version of Mar 17 1997 had some interesting and illuminating introductory historical remarks. They merit a reappearance in Appendix "Brief history of chaos", the "Old quantum theory section".

Now, something about the importance of this chapter to the book as a whole. As everybody should know, the chaotic collinear helium calculation is the prettiest physical application of the Gutzwiller-Voros Zeta function recycled.

In fall 1999 semester Cvitanović taught a pure Quantum Chaos course www.nbi.dk/ predrag/NUcourses/Fysik711B-99-sched.html
whose goal was to start with the Schrëdinger equation and end with the semiclassical quantization of collinear helium: it took

6 weeks to derive the Gutzwiller trace formula
3 weeks of random matrix theory
2 weeks of classical dynamics; periodic orbits of flows, escape rate, classical trace formula (by this time 1/2 kids can find periodic orbits of Roessler flow, their periods and stabilities)
2 weeks of mesoscopic physics (in the exercises I hope to get them to go through a derivation of a zeta function from the trace formula, substitute their data into the escape rate calculation, and see how far are they from escape rate = 0; start finding periodic orbits, stabilities of collinear helium)
2 weeks of zeta functions, shadowing (in the exercises they should get some estimate of helium spectrum, no matter how rough)
1 summary lecture

Cvitanović failed to reach the helium quantization, concluding that one should give up on it altogether; it is impossible to use as a textbook example.

Suppose they have understood how to integrate flows, look at Poincare sections, integrate Jacobians along flows, use Newton method, find periodic orbits, have some idea of symbolic dynamics, have classical and semiclassical cycle expansions, can they in 2 weeks of work get a few helium spectral lines out?

Should probably help them by telling them that the full analysis requires also $\sigma_p$ and $\alpha_p$, maybe say a few wise words of why are they what they are for the given cycles, then give them a table with all this information for all orbits up to length 5. They cannot cheat, as I expect to see their calculations.

I think that would be a humane approach - they would have learned something about semiclassical methods and chaotic dynamics, and come to appreciate that all this heavy machinery has good physical applications.

## 21.1 Collinear helium – the classical dynamics

The monodromy matrix in 3-$d$ is perhaps too hard, maybe have only a few words that in collinear case we are missing transverse oscillations and thus screwing up Maslov's and local quantum numbers.

## 21.2 Semiclassical quantization of collinear helium

Imagine yourself as a good graduate student, trying to reconstruct Tanner's PhD from this 3 and 1/2 page abstract. Even if you succeed, what do you compare to? is there enough information about periodic orbits, are the tables using all kinds of N,K,n, N,l, WKB that have no immediate connection to m and symmetric antisymmetric zeta's mentioned in the text?

Conclude with some wise words on intermittency.

### Historical comments

To: Douglas Stone, on 1917 Einstein paper (15 Sep 2005)

Thanks for delightful Physics Today article.

i talked about the paper to two people from that era about 15 years ago, Sir Rudolf Peierls and Bram Pais, and both knew nothing about the 1917 article. However, Theo Geisel has unearthed reference that shows that in early 20s (? I can chase that down) Born did have a study group meeting in his house that studied Poincaré's Mechanique Celeste.

But before Ian Percival included the topological phase, and Wintgen and students recycled the Helium atom, knowing Mechanique Celeste was not enough.

I gave a talk about Helium and cycle expansions in Seattle in 1994 to -inter alia- Hans Bethe, who loved it so much that after the talk he pulled me aside and we trotted over to his secret place: the best lunch on campus (Business School). I asked him: "Would Quantum Mechanics look different if in 1917 Bohr and Kramers et al. figured out how to use the helium classical 3-body dynamics to quantize helium?"

He was very annoyed. He said - in Bethe Deutschinglish, if you have talked to him you, you can do it yourself:

"It would not matter at all!"

## Resumé

## Exercises

# 22 Periodic orbit theory of diffraction

The goals of this chapter: can be brief: state that the full semiclassical trace is the sum of real cycles, complex cycles, cycles with diffractive connection terms in their weigths, and $\hbar$ corrections. Extracted from Copenhagen School collective works.

Whelan may also go through Andreas' diffraction chapters since he has some background on that topic.

## 22.1 Semiclassical creeping contributions

The goals of this chapter: The original Gutzwiller trace formula receives contributions from classical periodic orbits, and is clearly wrong near bifurcations, stability islands, diffractive scatterers. It misses tunneling and creeping. One of contributions of the Copenhagen School to the subject was a systematic incorporation of these effects, making it clear that the correct semiclassical trace formula is a sum over ALL saddles, real or complex, with diffractive contributions included in spirit of Keller, with connection coefficients computed from normal-form approximations to diffractive regions.

This chapter should describe the Keller + Copenhagen formulas in the case of creeping around 2-d billiard edges.

Next chapter describes the simplest (wedge) diffractive effects.

The tunneling chapter cannot be written yet - still a research topic.

What is missing in Das Buch is the answer to the next obvious question - what about the h-bar correction. Here Gaspard, Vattay and collaborators have provided a rather pretty complete answer. There is some of that discussion in the (preliminary) Noise chapter by Vattay.

In developping this theory, Wirzba's treatment was crucial all along the way - often the effects are exponentially small, and without Wirzba's high accuracy exact calculations one would not have trusted the approximations, or would have no way to gauge the accuracy of higher $\hbar$ corrections.

# 23 Diffraction distraction

The goals of this chapter: The original Gutzwiller trace formula receives contributions from classical periodic orbits, and is clearly missing diffractive effects, as they are NOT classical. This chapter teaches the ideal student how to include diffractive contributions in spirit of Keller, with connection coefficients computed from normal-form approximations to diffractive regions.

Niall Whelan is distracted, as he is 1) in love, and 2) has left physics (mid-July 2000) for a job at Scotiabank in Toronto as a "quant". These two facts are not completely uncorrelated. Anna and Niall have found a great place to live in a very cool part of town.

### 23.1 Quantum eavesdropping

### 23.2 An application

#### Resumé

#### Exercises

## 24 Why does it work?

Andreas and Hans Henrik should perhaps co-ordinate some parts, if Andreas writes some stuff on Fredholm dets in a Hilbert space set-up.

### 24.1 Curvature expansions: geometric picture

Hans Henrik appears not to have read this section. Needs work.

### 24.2 Analyticity of spectral determinants

### 24.3 Hyperbolic maps

**24.4 On importance of pruning** Hans Henrik appears not to have read this section. Needs work.

#### Resumé

#### Exercises

Hans Henrik has not prepared any exercises. Needs work.

# 25 Pruning dead itineraries

With a full time job and two small girls for Kai there is not much time for such things as writing this chapter. Besides, Kai would rather be skiing. Kai is out in the industrial and computer world, with daily life concerned more with RAM chips, A/D converters, instantiation of C++ classes, etc., then chaos. There is a Croatian proverb - "They can put salt on my tail" meaning they can put salt on my tail, but first they have to catch me. And now that Kai is out of academia, he is hard to catch. But if he were to get into an inspired mood, nobody would contribute as much as Kai can when it comes to symbolic dynamics and pruning...

Predrag to Ralf Gekle, Stuttgart (8 Oct 2004):

I would like you to use his well-ordered symbol-plane.

Why? This is the ordering that clearly reveals the origin of the pruning front. It depends on the signs of expanding eigenvalues of the 3 basic types of motion incoded by the ternary symbolic dynamics: if the next symbol is oredr preserving, ternary tree is unchanges - if it is order reversing, the corresponding subbranch of the ternary tree is flipped. I expect these signs to be the same for 4-disk, hyperbola billiard and for your potential - the difference is that for for sufficiently low E you start getting (and losing again) islands of ellipticity along the pruning front. This is how

P. Dahlqvist and G. Russberg, "Existence of stable orbits in the $x^2y^2$ potential" *Phys. Rev. Lett.* **65**, 2837 (1990).

found the island of stability that disproved the long-standing conjecture that $x^2y^2$ (with E=0 in your case) potential is fully ergodic.

Explain the pruning front as on p. 221 of Hansen's thesis. The pruning arises as in Fig 10.4. If there is some other pruning mechanism, other than Figs 7.4 to 7.4, that would be very interesting. It could happen, as your effective potential has a bump ccoming up form the central "$dz$" region.

There seem to be 3 refs, I think I like the Nonlinearity one the best (if you read them, let me know which one you like the best).

F. Christiansen and A. Politi, A generating partition for the standard map Phys. Rev. E. 51, 3811 (1995). arXiv/chao-dyn/9411005

F. Christiansen and A. Politi, Symbolic encoding in symplectic maps Nonlinearity, 9, 1623 (1996).

F. Christiansen and A. Politi, Guidelines for the construction of a generating partition in the standard map Physica D, 109, 32 (1997).

## 25.1 Pruning front

### 28.5 Geometrization

#### Resumé

#### Exercises

## 29 Limbo

### 29.1 Intermittency

### 29.2 Phase transitions

### 29.3 Period doubling renormalization

### 29.4 Scaling, presentation functions

#### Resumé

## Summary and conclusions

## A A brief history of chaos

Andrey Shilnikov, 17 Sep 2005:

It irks me to no end that the public (in large part because of Gleick) thinks the damn subject is called chaos theory. I tell people I study nonlinear dynamics and they have no clue what I mean. I then say it's popularly called chaos theory and they then think they know what I do. Of course, the term "complex systems" is even worse, but what can one do.

Obviously I don't look at chaos being on position of an "ergodist" or "Chaotians."

In skimming through the article, I see tons of familiar names and the annoying term "chaologist" (used multiple times). Of course, it's not as bad as Frank Moon's "Chaotians."

I sort of guess that you do not really want to focus on that part of the theory, however you have added Pesin' comments to the appendix, etc what for..?

Porter:

There was somebody who interviewed in the philosophy dept at Cornell when I was in grad school who talked about the history of chaos. Maybe it was Aubin? In any event, the talk that was given was horribly wrong on several points—not that I actually remember what those points are (except for his extremely annoying comment that nonlinear dynamics was dead and that all the special groups associated with it would soon disappear).

I glanced through the current version of this appendix. I think you should add something about the FPU problem (perhaps in a small note). In particular, its use in the first application of the "overlap criterion" (in the paper by Chirikov and Izrailev) and Joe Ford's work around the same time seems very relevant in chaos's history. I mention it in my review article in the 3/05 *Chaos* focus issue, but there are at least one or two other articles from that issue written by Chirikov, Izrailev, and one or two others that give more historical context on that particular aspect of FPU. All the info you'd actually need is most likely in that FPU book you have, but it might be interesting to see what's in the new articles.

I think you should have some remark on mixed regular-chaotic dynamics in your book. This would be an appropriate appendix, for instance, because I think an intro grad student can go through the entire book and not realize that the mixed case is the one they should expect if one pulls a "random" low-dimensional system out of a hat. One of the faults I've seen in courses I took and in monographs I've read is that even when they do a good job of presenting certain problems, they don't give the right impression that such problems have very small or zero measure in the set of all problems people might encounter if they're doing physics or applied math [or other subjects].

A section on dissipative systems would also be appropriate here. You could also mention some of the physical ones, such as the experimental work on convection.

I've occasionally used the term "dynamicist" for myself after I've heard the mathematicians in the room say I'm a physicist and the physicists say that I'm a mathematician.

Beginning of my http://www.mathstat.gsu.edu/ matals/research/sync.pdf is on topic.

Speaking of chaos (in Russia, in particular), I do not see why billiards and hyperbolic systems should be the only representative examples of the chaos theory, in addition to a unimodal map. How about routes to chaos in dissipative systems? Like the Lorenz equation ( I mean here Guckenheimer-Williams and Afraimovich-Bykov-Shilnikov explanations), torus breakdown, saddle-focus (Shilnikov 1965), etc an avalanche of excellent works done in Gorky. Besides - Zaslavsky, Chirikov (Novossibirsk), Sharkovsky (Kiev). Those folks deserved some credit.

IMHO - without those an intro into chaos is misleading, like sync and synchronization.

Leonid Shilnikov, 5 April 2000, *Homoclinic orbits: since Poincaré till today.*
http://www.wias-berlin.de/publications/preprints/571/wias_preprints_571.pdf

### A.1 Chaos is born

### A.2 Chaos grows up

### A.3 Chaos with us

### A.4 Death of the Old Quantum Theory

Tanner's "Helium atom", version of Mar 17 1997 had some interesting and illuminatingi introductory historical remarks. They merit a reappearance in this section.

### A.5 Brief history of $\pi/2$

## B Billiards

### B.1 One-dimensional maps

### B.2 Linear fractional maps

### B.3 Billiards with gravity: 2 balls on a line

### B.4 Wedge billiard

### B.5 Transfer operators

## C Linear stability of Hamiltonian flows

### C.1 Symplectic invariance

### C.2 Monodromy matrix for Hamiltonian flows

Whelan does not like this section. He thinks about the propagation of monodromy matrices very differently and has a better way of expressing it which looks less technical and appeals more to intuition and geometry. (Learned from Creagh.) Hopefully he will write that up here.

# D Implementing evolution

## D.1 Material invariants

## D.2 Implementing evolution

### Exercises

# E Symbolic dynamics techniques

## E.1 Symbolic dynamics, basic notions

## E.2 Topological zeta functions for infinite subshifts

Marco.Goetz@mie.alcatel.be asks:

1. Is there an efficient algorithm for generating a symbolic prime cycle list, even if more than two symbols are needed for cycle coding?
2. Can one download prime cycle lists anywhere ?

Predrag wrote a Mathematica program which parses all possible finite symbol strings of increasing length, cyclically permutes them to the cycle point with lowest lexical ordering, keeps one cycle point per cycle, and then check whether the sequence is a repeat of a subsequence.

Thomas Schreiber has generated a large number of prime cycles for Lorentz gas, where there are 11 symbols and complicated pruning. The largest prime cycle set was probably Schreiber's - millions of prime cycles. M. Sieber had generated millions of cycles for his hyperbolic billiard PhD thesis. The longest prime cycles were probably generated by Grassberger, he computed Henon topological entropy to cycle length 32 or so.

Kai Hansen has generated prime cyles for many different problems.

Cycle counting of sect. 13.5.2 is useful for checking that all prime cycles have been found.

There might exist a nice prime factorization theorem to prove which might also be useful in generating prime cycles. In appendix F.3 it is written down only for complete binary symbolic dynamics, but it might generalize to any finite subshift.

As far as authors know, there is no site from which one can download prime cycle lists. It is probably easier to generate them from the scratch.

## E.3 Pruned Bernoulli shift

## E.4 Prime factorization for dynamical itineraries

## E.5 Möbius inversion formula, ordered sets

## E.6 Counting curvatures

### Exercises

# F Discrete symmetries

## F.1 $C_{4v}$ factorization

## F.2 $C_{2v}$ factorization

## F.3 Symmetries of the symbol plane

# G Statistical mechanics applications

## G.1 Diffusion in sawtooth and cat maps

### Exercises

# H Infinite dimensional operators

## H.1 Matrix-valued functions

## H.2 Trace class and Hilbert-Schmidt class

### H.3 Determinants of trace class operators

### H.4 Von Koch matrices

### H.5 Regularization

## I Diffraction distraction

## J Quantum and noise trace formulas

Regrettably, DasBuch omits any serious discussion of the Martin-Siggia-Rose type weak noise corrections. It could have included the general blah-blah of *Chaotic Field Theory: a Sketch - invited talk, Dynamics Days Asia-Pacific, 13 - 16 July, 1999*, to be published in *Physica* **D**. This paper discusses the possible ways for improvement of diagrammatic summation in chaotic field theories. The author develops an original viewpoint according to which this can be done with the help of periodic orbit approach. The results are new and original. Personally, I do not think that such an approach will be succesful since quantum many-body effects cannot be reduced only to simple periodic orbits. However, this approach can be fruitful in other contexts.

The details are in the three papers:

Spectrum of stochastic evolution operators: Local matrix representation approach, - (with C.P. Dettmann, G. Palla, N. Søndergaard and G. Vattay), chao-dyn/9904027; Phys. Rev. E 60, 3936 (1999).

Trace formulas for stochastic evolution operators: Smooth conjugation method, (with C.P. Dettmann, R. Mainieri and G. Vattay), chao-dyn/9811003; Nonlinearity 12, 939 (1999).

Trace formulas for stochastic evolution operators: Weak noise perturbation theory, (with C.P. Dettmann, R. Mainieri and G. Vattay), chao-dyn/9807034, J. Stat. Phys. 93, 981 (1998).

As far as one can tell, no one has evaluated such corrections before, strange as it seems. Probably because with noise one is so unsure about nature of the noise itself that $n$th order correction is beyond the point.

Doing continuous time flows seemed to require the same kind of corrections, with diagrams standing for integrals rather than sums - as it was, only Dettmann was able to evaluate higher order graphs, so no one ever tried weakly stochastic flows in continuous time.

### J.1 Noise and quantum corrections to trace formulas

# K Quasiclassics, for a general potential

# L Solutions

This material could indeed be kept on the Web - if a student is sufficiently motivated to start working on a problem, shw will be sufficiently motivated to click on the book homepage.

My suggestion in some larger context is to refer to prof. David Donoho's work on wavelets (donoho@playfair.stanford.edu); Donoho's quest for "reproducible research" (regarding which he has a very enjoyable paper on line) is to provide all interested parties the actual code to reproduce all figures in any of his papers, reports, talks, etc.. He has chosen to do this in matlab with the base being his freely available (over internet) Wavelab software. I would really like the zeta function research group at large – Copenhagen group, Steiner's group, Degli Esposti's group, Baladi's group, Gaspard's group, Ruelle, Hejhal, the Orsay group, Berry's group, Keating, Voros, Eckhardt, etc. to strive to do the same; would it not be great to have many examples like the xpinball software for other researchers, students, etc. to learn from? I would like the eigenvalue data or prime orbit data from Steiner's work or Hejhal's work. Why not baker's and cat's maps on line? It is really a waste of effort to have to reproduce the code that others have developed to understand in detail th meaning of some figure in a book or article, when the goal is not coding but physics.

# M Projects

This material should be kept on the book hompage, not in the paper version.

## M.1 Deterministic diffusion, zig-zag map

## M.2 Deterministic diffusion, sawtooth map

## M.3 Cardioid Billiard

## M.4 Ray splitting billiard

## M.5 Hopf's last hope

## M.6 Hydrogen in external magnetic field - disociation rate

## M.7 Product of random matrices

## Bibliography

## Index

Currently the index is a big mess; both incomplete and in need of much editing.

For editing purposes, Predrag has generated full tabulation of words used.

One might want to create author index.

## Das Buch: In memoriam

Caro Professore Cvitanović and friends,

It is with a slight sadness in my heart and some water in my eyes che io have to write this letter. I have read with great care all electronic mail of recent time and I feel that there is a great equivoco. Il Libro has been enriched with your contributions and I am indebted to you and your families for your labors. BUT that does not make you the author.

E' una tradizione di scrivere un libro, specialy a scientific book, that the author use articles of science that have been publised for explaining the subject. I have, with full citations and my greatest thanks, used your work, but it is me who has organized, collected and re-explained all the work in one coherent whole. I hope that you can see that it is not a simple lavoro.

So, per favore, stop claiming the authorship.

Inquietum est cor nostrum, donec requiescat in te.

Commendatore, Dottore e Professore Gatto Nero

PS: Sorry for the English, my secretary is visiting her family in New Zeland.

Universita Cattolica del Sacro Cuore 8 Noviembre 1999
Dipartimento di Matematica Applicata
Cittá del Vaticano

Inventas vitam iuvat excoluisse per artes.

### Comments by Professore R. Artuso

The inscription reads:

Inventas vitam juvat excoluisse per artes

loosely translated "And they who bettered life on earth by new found mastery." (Word for word: inventions enhance life which is beautified through art.) The words are taken from Vergilius Aeneid, the 6th song, verse 663;

The text is not universally accepted, unlike the quote:
cfr http://vergil.classics.upenn.edu/home/

"Inquietum est cor nostrum usque ad requiescat in te" - "Restless is our heart until it rests in you", said the church father Augustinus.
http://ccat.sas.upenn.edu/jod/latinconf/1.html (magna virtus tua, et sapientiae tuae non est numerus, just to stay with Augustinus)