Exercises

Exercise 36.1 Monodromy matrix from second variations of the action. *Show that*

$$D_{\perp j}/D'_{\perp j} = (\mathbf{1} - M) \tag{36.26}$$

Exercise 36.2 Jacobi gymnastics. *Prove that the ratio of determinants in (R.70) can be expressed as*

$$\frac{\det D'_{\perp j}(q_\parallel, 0, E)}{\det D_{\perp j}(q_\parallel, 0, E)} = \det \begin{pmatrix} I - M_{qq} & -M_{qp} \\ -M_{pq} & I - M_{pp} \end{pmatrix} = \det (1 - M_j)\,, \tag{36.27}$$

where M_j is the monodromy matrix of the periodic orbit.

Exercise 36.3 Volume of d-dimensional sphere. *Show that the volume of a d-dimensional sphere of radius r equals $\pi^{d/2} r^d / \Gamma(1 + d/2)$. Show that $\Gamma(1 + d/2) = \Gamma(d/2)d/2$.*

Exercise 36.4 Average density of states in 1 dimension. *Show that in one dimension the average density of states is given by (36.8)*

$$\bar{d}(E) = \frac{T(E)}{2\pi\hbar},$$

where $T(E)$ is the time period of the 1-dimensional motion and show that

$$\bar{N}(E) = \frac{S(E)}{2\pi\hbar}\,, \tag{36.28}$$

where $S(E) = \oint p(q, E)\, dq$ is the action of the orbit.

Exercise 36.5 Average density of states in 2 dimensions. *Show that in 2 dimensions the average density of states is given by (36.9)*

$$\bar{d}(E) = \frac{m\mathcal{A}(E)}{2\pi\hbar^2}\,,$$

where $\mathcal{A}(E)$ is the classically allowed area of configuration space for which $U(q) < E$.

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[5] Dorte: not sure how to appendix this - and the fax not very clear, so you are on your own!! ⇓PRELIMINARY

Exercise 36.6 2-d Hamiltonian flows. *The simplest application of (??) is to 2-dimensional hyperbolic Hamiltonian flows. In this case the derivation of the quasiclassical trace formula is very simple. The [2×2]-dimensional phase space fundamental matrix is*

$$\begin{pmatrix} \delta q' \\ \delta p' \end{pmatrix} = M \begin{pmatrix} \delta q \\ \delta p \end{pmatrix} .$$

In 2-d phase space the curvature matrix $\mathbf{M}$ is $[1 \times 1]$, ie. the scalar $\kappa := \partial p/\partial q$. Divide through with δq.

$$\frac{dq'}{dq} \begin{pmatrix} 1 \\ \kappa' \end{pmatrix} = M \begin{pmatrix} 1 \\ \kappa \end{pmatrix} . \tag{36.29}$$

Writing $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ we have

$$\frac{dq'}{dq} = A + B\kappa \,, \qquad \kappa' = M(\kappa) = \frac{A+B\kappa}{C+D\kappa} \,.$$

For a periodic orbit there are two solutions

$$M \begin{pmatrix} 1 \\ \kappa_\pm \end{pmatrix} = \Lambda_\pm \begin{pmatrix} 1 \\ \kappa_\pm \end{pmatrix}$$

so the configuration volume ratio $\frac{dq'}{dq}$ is for a periodic orbit simply the eigenvalue of the phase space Jacobian. The κ part of the trace is

$$\int d\kappa \, \delta(\kappa - M(\kappa)) = \frac{1}{\left|1 - \frac{\partial M}{\partial \kappa}\right|} \,. \qquad \frac{\partial M}{\partial \kappa} = \frac{1}{(A+B\kappa)^2} = \frac{1}{\Lambda_\pm^2}$$

so the curvature matrix contribution to the trace is

$$\int d\kappa \, \delta(\kappa - M(\kappa)) \left| \frac{dq'}{dq} \right|^{1/2} = \frac{|\Lambda_+|^{1/2}}{|1 - 1/\Lambda_+^2|} + \frac{|\Lambda_-|^{1/2}}{|1 - 1/\Lambda_-^2|}$$

Substituting into (15.7) yields the spectral determinant

$$\begin{aligned} \det(\mathbf{1} - \mathcal{L}) &= \exp\left(- \sum_{p,r} \frac{1}{r \mid \Lambda_p^r \mid} \frac{e^{irS_p(E) + itopologicalp\pi}}{(1 - 1/\Lambda_p^r)^2} \Delta_{p,r} \right) \\ \Delta_{p,r} &= \frac{\mid \Lambda_p^r \mid^{1/2}}{1 - 1/\Lambda_p^{2r}} + \frac{\mid \Lambda_p^r \mid^{-5/2}}{1 - 1/\Lambda_p^{2r}} \,, \end{aligned} \tag{36.30}$$

⇑PRELIMINARY