symmetry!dynamical system
dynamics!symmetry

Exercises

Exercise 23.1 **To be constructed:** *Derive the trace formula (??) for a discrete symmetry*

A dynamical system $(\mathcal{M}, f)$ is invariant under a symmetry group G if the map $f : \mathcal{M} \to \mathcal{M}$ (or the continous flow f^t) from the d-dimensional manifold $\mathcal{M}$ into itself (with d finite) is invariant

$$f(\mathbf{g}x) = \mathbf{g}f(x)$$

for any coordinate $x \in \mathcal{M}$ and any finite non-singular linear representation $\mathbf{g}$ of any element $g \in G$.

Show that the evolution operator *$\mathcal{L}(x, y)$ (23.1) satisfies (23.25):*

$$\mathcal{L}(x, y) = \mathcal{L}(\mathbf{g}x, \mathbf{g}y).\,,$$

Solution: (move to solutions!)
Rotate coordinates x′= $\mathbf{g}$x:

$$\mathcal{L}(x', y') = \delta(\mathbf{g}y - f(\mathbf{g}x)). = |\det \mathbf{g}|^{-1} \delta(y - f(x)). = \mathcal{L}(x, y) = |\det \mathbf{g}| \, \mathcal{L}(\mathbf{g}x, \mathbf{g}y)..$$

For a compact semisimple Lie group $|\det \mathbf{g}| = 1$, hence (23.25).

Exercise 23.2 **Relative periodic orbits for circles, bagels and spheres:** *(a) Show that relative periodic orbits for a point scattering specularly in a circular billiard are single scattering arcs. Compute their stability. Compute the spectrum.*

(b) Show that relative periodic orbits for a point scattering specularly in the plane that slices symmetrically upper half of a bagel (floating tire, torus) are single scattering arcs. Compute their stability. Compute the spectrum. Compute the escape rate.

(c) Show that relative periodic orbits for a point scattering specularly within a sphere billiard are single scattering arcs. Compute their stability, spectrum.