three-disk@3-disk!pruning front
pruning front!3-disk

Exercises

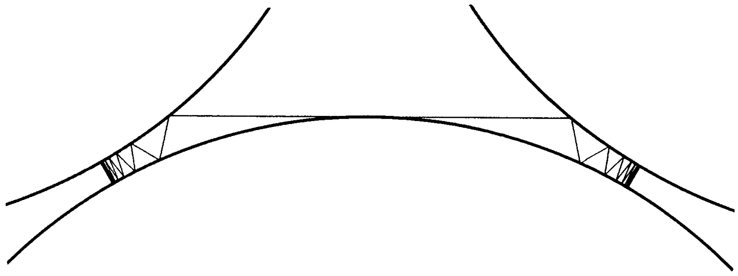
31.1 Alphabet {0,1}, prune only the fixed point $\overline{0}$. This is equivalent to the *infinite* alphabet $\{1, 2, 3, 4, \ldots\}$ unrestricted symbolic dynamics. The prime cycles are labelled by all non-repeating sequences of integers, ordered lexically: $t_n, n > 0$; $t_{mn}, t_{mmn}, \ldots, n > m > 0$; $t_{mnr}, r > n > m > 0, \ldots$ (see table **??**). Now the number of fundamental cycles is infinite as well:

$$
\begin{aligned}
1/\zeta \;=\; & 1 - \sum_{n>0} t_n - \sum_{n>m>0} (t_{mn} - t_n t_m) \\
& - \sum_{n>m>0} (t_{mmn} - t_m t_{mn}) - \sum_{n>m>0} (t_{mnn} - t_{mn} t_n) \qquad (31.1) \\
& - \sum_{r>n>m>0} (t_{mnr} + t_{mrn} - t_{mn} t_r - t_{mr} t_n - t_m t_{nr} + t_m t_n t_r) \cdots \qquad (31.2)
\end{aligned}
$$

z˙gauss

We have already encountered this sum in the Ulam map cycle expansion in sect. ?!. As we shall see in sect. ?!, this grammar plays an important role in description of fixed points of marginal stability.

31.2 3-disk pruning (Not easy) Show that for 3-disk game of pinball the pruning of orbits starts at $R : a = 2.04821419\ldots$.



(Kai T. Hansen)

31.3 Alphabet {0,1}, prune _1000_, _00100_, _01100_. This example is motivated by the pruning front description of the symbolic dynamics for the Hénon-type maps, sect. 31.2.

step 1. _1000_ prunes all cycles with a _000_ subsequence with the exception of the fixed point $\overline{0}$; hence we factor out $(1 - t_0)$ explicitly, and prune _000_ from the rest. This means that x_0 is an isolated fixed point - no cycle stays in its vicinity for more than 2 iterations. In the notation of the example 2!?, the alphabet is $\{1, 2, 3; \overline{0}\}$, and the remaining pruning rules have to be rewritten in terms of symbols 2=10, 3=100:

restore right reference

step 2. alphabet $\{1, 2, 3; \overline{0}\}$, prune _33_, _213_,_313_. This means that the 3-cycle $\overline{3} = \overline{100}$ is pruned and no long cycles stay close enough to it for a single _100_ repeat.

ight reference

As in example 1?!, prohibition of _33_ is implemented by dropping the symbol "3" and extending the alphabet by the allowed blocks 13, 23:

step 3. alphabet $\{1,\, 2,\, \underline{13},\, \underline{23};\, \overline{0}\}$, prune $\_2\underline{13}\_$, $\_\underline{23}\,\underline{13}\_$, $\_\underline{13}\,\underline{13}\_$, where $\underline{13} = 13$, $\underline{23} = 23$ are now used as single letters. Pruning of the repetitions $\_\underline{13}\,\underline{13}\_$ (the 4-cycle $\overline{\underline{13}} = \overline{1100}$ is pruned) yields the

result: alphabet $\{1,\, 2,\, \underline{23},\, \underline{113};\, \overline{0}\}$, unrestricted 4-ary dynamics. The other remaining possible blocks $\_\underline{213}\_$, $\_\underline{2313}\_$ are forbidden by the rules of step 3. The cycle expansion is given by

$$1/\zeta = (1 - t_0)(1 - t_1 - t_2 - t_{23} - t_{113}) \tag{31.3}$$

hen1a

for unrestricted 4-letter alphabet $\{1,\, 2,\, \underline{23},\, \underline{113}\}$.

add Markov graph exercises.