Galilean invariance
invariance!Galilean
smooth!dynamical system
dynamical system!infinite

Exercises

Exercise 25.1 **Galilean invariance of the Kuramoto-Sivashinsky equation.**

(a) *Verify that the Kuramoto-Sivashinsky equation is Galilean invariant: if $u(x,t)$ is a solution, then $v + u(x + 2vt, t)$, with v an arbitrary constant velocity, i s also a solution.*

(b) *Verify that mean*

$$\langle u \rangle = \frac{1}{L} \int_L dx\, u$$

is conserved by the flow.

(c) *Argue that the choice (25.5) of the vanishing mean velocity, $\langle u \rangle = 0$ leads to no loss of generality in calculations that follow.*

(d) *[thinking is extra cost] Inspection of various "turbulent" solutions of Kuramoto-Sivashinsky equation reveals subregions of "traveling waves" with locally nonzero $\langle u \rangle$. Is there a way to use Galilean invariance locally, even though we eliminated it by the $\langle u \rangle = 0$ condition?*

Exercise 25.2 **Infinite dimensional dynamical systems are not smooth.** *Many of the operations we consider natural for finite dimensional systems do not have smooth behavior in infinite dimensional vector spaces. Consider, as an example, a concentration ϕ diffusing on $\mathbb{R}$ according to the diffusion equation*

$$\partial_t \phi = \frac{1}{2} \nabla^2 \phi \,.$$

(a) *Interpret the partial differential equation as an infinite dimensional dynamical system. That is, write it as $\dot{x} = F(x)$ and find the velocity field.*

(b) *Show by examining the norm*

$$\|\phi\|^2 = \int_{\mathbb{R}} dx\, \phi^2(x)$$

that the vector field F is not continuous.

(c) *Try the norm*

$$\|\phi\| = \sup_{x \in \mathbb{R}} |\phi(x)| \,.$$

Is F continuous?

(d) *Argue that the semi-flow nature of the problem is not the cause of our difficulties.*

(e) *Do you see a way of generalizing these results?*