Exercises

three-disk@3-disk!cycle!analytically
stability!exact

Exercise 8.1 **Fundamental domain fixed points.** *Use the formula (6.11) for billiard fundamental matrix to compute the periods T_p and the expanding eigenvalues Λ_p of the fundamental domain $\overline{0}$ (the 2-cycle of the complete 3-disk space) and $\overline{1}$ (the 3-cycle of the complete 3-disk space) fixed points:*

	T_p	Λ_p
$\overline{0}$:	$R-2$	$R-1+R\sqrt{1-2/R}$
$\overline{1}$:	$R-\sqrt{3}$	$-\frac{2R}{\sqrt{3}}+1-\frac{2R}{\sqrt{3}}\sqrt{1-\sqrt{3}/R}$

(8.21)

We have set the disk radius to $a=1$.

Exercise 8.2 **Fundamental domain 2-cycle.** *Verify that for the $\overline{10}$-cycle the cycle length and the trace of the fundamental matrix are given by*

$$\begin{aligned} L_{10} &= 2\sqrt{R^2-\sqrt{3}R+1}-2, \\ \operatorname{tr}\mathbf{J}_{10} &= 2L_{10}+2+\frac{1}{2}\,\frac{L_{10}(L_{10}+2)^2}{\sqrt{3}R/2-1}\ . \end{aligned} \tag{8.22}$$

The $\overline{10}$-cycle is drawn in figure 11.6. The unstable eigenvalue Λ_{10} follows from (4.19).

recheck - fixed up by Per

⇓PRIVATE

Exercise 8.3 **Driven damped harmonic oscillator limit cycle.** *Driven damped harmonic oscillator equations of motion are:*

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Exercise 8.4 **A limit cycle with analytic stability exponent.** *There are only two examples of nonlinear flows for which the stability eigenvalues can be evaluated analytically. Both are cheats. One example is the 2-d flow*

$$\dot{q}=\ p+q(1-q^2-p^2)\,, \qquad \dot{p}=-q+p(1-q^2-p^2)\,. \tag{8.23}$$

Determine all periodic solutions of this flow, and determine analytically their stability exponents. Hint: go to polar coordinates $(q,p)=(r\cos\theta, r\sin\theta)$.

G. Bard Ermentrout

Exercise 8.5 **The other example of a limit cycle with analytic stability exponent.** *What is the other example of a nonlinear flow for which the stability eigenvalues can be evaluated analytically? Hint: email G.B. Ermentrout.*

Exercise 8.6 **Yet another example of a limit cycle with analytic stability exponent.** *Prove G.B. Ermentrout wrong by solving a third example (or more) of a nonlinear flow for which the stability eigenvalues can be evaluated analytically.*