## Exercises

**Exercise L.1 Recursion relations.** *Verify that the following recursione relations are satisfied*

$$u_{k+2} = \chi u_{k+1} - u_k$$

*where* $u_k = A_k, B_k, C_k, D_k$.

**Exercise L.2 Arnol'd cat map.** *Show that for* $\chi = 3$, $A_k = F_{2k+1}$, $B_k = L_{2k+1}$, $C_k = L_{2k}$ *and* $D_k = 5F_{2k}$, *where* $F_n$ *and* $L_n$ *are the Fibonacci and Lucas numbers..*

**Exercise L.3 Pruning rules for substrings of length 2.** *Take* $K = 8$ *and draw the region determined by (L.5).*

**Exercise L.4 Diagonalization of $A$.** *Show that* $A$ *can be diagonalized by considering the auxiliary matrix* $U$

$$U \stackrel{\text{def}}{=} \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \epsilon_0 & \epsilon_1 & \cdots & \epsilon_{T-2} & \epsilon_{T-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \epsilon_0^{T-2} & \epsilon_1^{T-2} & \cdots & \epsilon_{T-2}^{T-2} & \epsilon_{T-1}^{T-2} \\ \epsilon_0^{T-1} & \epsilon_1^{T-1} & \cdots & \epsilon_{T-2}^{T-1} & \epsilon_{T-1}^{T-1} \end{pmatrix} .$$

*In fact* $U^{-1}AU$ *is a diagonal matrix (the diagonal elements coinciding with* $f(\epsilon_j)$*).*

**Exercise L.5 Periodic points of cat maps.** *Verify that (L.18) is is exactly the number of* $T$*–periodic points of the map when* $K$ *is an integer.*

**Exercise L.6 Probability distribution.** *Higher order moments can be computed easily for integer* $K$ *(or generic* $K$ *within the quasilinear approximation), by generalizations of (L.20): show that the results prove that, given a period* $T$, *the distribution of periodic orbits with respect to their jumping number is asymptotically Gaussian, with parameter* $D^{(q.l.)}$.