Exercises

⇓PRELIMINARY

Exercise E.1 Perron-Frobenius operator is the adjoint of the Koopman operator. *Check (??) - it might be wrong as it stands. Pay attention to presence/absence of a Jacobian.*

⇑PRELIMINARY

Exercise E.2 Exponential form of semigroup elements. *Check that the Koopman operator and the evolution generator commute, $\mathcal{K}^t \mathcal{A} = \mathcal{A}\mathcal{K}^t$, by considering the action of both operators on an arbitrary state space function $a(x)$.*

Exercise E.3 Symplectic volume preservation. *Check that the sequence of mappings (E.14) is volume preserving, $\det \hat{U} = 1$.*

Exercise E.4 Non-commutativity. *Check that the commutators in (E.12) are not vanishing by showing that*

$$[\mathcal{B}, \mathcal{C}] = -p \left(V'' \frac{\partial}{\partial p} - V' \frac{\partial}{\partial q} \right) .$$

this might be an obsolete version

Exercise E.5 Symplectic leapfrog integrator. *Implement (E.15) for 2-dimensional Hamiltonian flows; compare it with Runge-Kutta integrator by integrating trajectories in some (chaotic) Hamiltonian flow.*