Exercises

three-disk@3-disk!cycle!count

Exercise G.1 Lefschetz zeta function. *Elucidate the relation betveen the topological zeta function and the Lefschetz zeta function. This should help make sense of sect. F.4.* for Vachtang

⇓PRELIMINARY

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Exercise G.2 Counting the 3-disk pinball counterterms. *Verify that the number of terms in the 3-disk pinball curvature expansion (18.36) is given by*

$$\begin{aligned}\prod_p (1+t_p) &= \frac{1-3z^4-2z^6}{1-3z^2-2z^3} = 1+3z^2+2z^3+\frac{z^4(6+12z+2z^2)}{1-3z^2-2z^3} \\ &= 1+3z^2+2z^3+6z^4+12z^5+20z^6+48z^7+84z^8+184z^9 \quad \text{(G.3)}.\end{aligned}$$

This means that, for example, c_6 has a total of 20 terms, in agreement with the explicit 3-disk cycle expansion (18.37).

Exercise G.3 Cycle expansion denominators****.** *Prove that the denominator of c_k is indeed D_k, as asserted (F.23).*

⇓PRELIMINARY

Exercise G.4 Möbius inversion****.** *Do sect. F.3.1 right; prove prime factorization of symbolic strings for arbitrary grammar, not just complete binary, explain how this partitions the Poincaré section into regions shadowed by particular sequences of pseudocycles.*

Exercise G.5 Factorizations. *do some factorizations by hand...*

⇑PRELIMINARY

Exercise G.6 Counting subsets of cycles. *The techniques developed above can be generalized to counting subsets of cycles. Consider the simplest example of a dynamical system with a complete binary tree, a repeller map (11.8) with two straight branches, which we label 0 and 1. Every cycle weight for such map factorizes, with a factor t_0 for each 0, and factor t_1 for each 1 in its symbol string. The transition matrix traces (13.5) collapse to $tr(T^k) = (t_0+t_1)^k$, and $1/\zeta$ is simply* derive this!

$$\prod_p (1-t_p) = 1-t_0-t_1 \qquad \text{(G.4)}$$

Substituting into the identity

$$\prod_p (1+t_p) = \prod_p \frac{1-{t_p}^2}{1-t_p}$$

we obtain

$$\begin{aligned}\prod_p (1+t_p) &= \frac{1-t_0^2-t_1^2}{1-t_0-t_1} = 1+t_0+t_1+\frac{2t_0t_1}{1-t_0-t_1} \\ &= 1+t_0+t_1+\sum_{n=2}^{\infty}\sum_{k=1}^{n-1} 2\binom{n-2}{k-1} t_0^k t_1^{n-k}. \qquad \text{(G.5)}\end{aligned}$$

Hence for $n \geq 2$ the number of terms in the expansion ?! with k 0's and $n-k$ 1's in their symbol sequences is $2\binom{n-2}{k-1}$. This is the degeneracy of distinct cycle eigenvalues in fig.?!; for systems with non-uniform hyperbolicity this degeneracy is lifted (see fig. ?!). copy from AAC

In order to count the number of prime cycles in each such subset we denote with $M_{n,k}$ $(n = 1, 2, \ldots ; k = \{0, 1\}$ for $n = 1$; $k = 1, \ldots, n-1$ for $n \geq 2)$ the number of prime n-cycles whose labels contain k zeros, use binomial string counting and Möbius inversion and obtain

$$\begin{aligned} M_{1,0} &= M_{1,1} = 1 \\ nM_{n,k} &= \sum_{m\left|\frac{n}{k}\right.} \mu(m) \binom{n/m}{k/m}, \quad n \geq 2\,, k = 1, \ldots, n-1 \end{aligned}$$

where the sum is over all m which divide both n and k.