Exercises

Exercise I.1 Stretching factor. *Prove the multiplicative property of the stretching factor (I.2). Why should we extend the phase space with the tangent space?*

Exercise I.2 Dynamo rate. *Suppose that the fluid dynamics is highly dissipative and can be well approximated by the piecewise linear map*

$$f(x) = \begin{cases} 1 + ax & \textit{if} \quad x < 0, \\ 1 - bx & \textit{if} \quad x > 0, \end{cases} \tag{I.37}$$

on an appropriate surface of section ($a, b > 2$). *Suppose also that the return time is constant* T_a *for* $x < 0$ *and* T_b *for* $x > 0$. *Show that the dynamo zeta is*

$$1/\zeta_{dyn}(s) = 1 - e^{sT_a} + e^{sT_b}. \tag{I.38}$$

Show also that the escape rate is the leading zero of

$$1/\zeta_0(s) = 1 - e^{sT_a}/a - e^{sT_b}/b. \tag{I.39}$$

Calculate the dynamo and the escape rates analytically if $b = a^2$ *and* $T_b = 2T_a$. *Do the calculation for the case when you reverse the signs of the slopes of the map. What is the difference?*