Chapter 42

Periodic orbit theory of diffraction

You know that love
Will creep in service when it cannot go.
William Shakespeare: Two Gentlemen of Verona, act iv. sc. 2.

(Andreas Wirzba)

We will cast here the leading quantum diffraction effects in scattering into periodic orbit form, by including into the cycle expansions complex saddles as well.

We start by a brief review of the elastic scattering of a point particle from finite collection of non-overlapping scattering regions in terms of the standard textbook scattering theory, and then develop Keller's semiclassical diffraction theory for scattering off n disks in a plane.

42.1 Semiclassical creeping contributions

Children learne to creepe ere they can learne to goe
John Heywood, Proverbes 1546

For periodic orbits with creeping sections [38.23, ?, ?, ?, ?] the symbolic alphabet has to be extended. Furthermore, depending on the geometry, there might be nontrivial pruning rules based on the so called ghost orbits, see refs. [38.12, 38.24].

If the creeping contributions are taken into account, the symbolic dynamics has to be generalized from single-letter symbols $\{j_i\}$ to triple-letter symbols $\{j_i, s_i \times \ell_i\}$ with $\ell_i \geq 1$ integer valued and $s_i = 0, \pm 1$ [1] By definition, the

[1] Actually, these are double-letter symbols as s_i and l_i are only counted as a product.

value $s_i = 0$ represents the non-creeping case, such that $\{j_i, 0 \times \ell_i\} = \{j_i, 0\} = \{j_i\}$ reduces to the old single-letter symbol. The magnitude of a nonzero ℓ_i corresponds to creeping sections of mode number $|\ell_i|$, whereas the sign $s_i = \pm 1$ signals whether the creeping path turns around the disk j_i in the positive or negative sense. Additional full creeping turns around a disk j' can be summed up as a geometrical series; therefore they do not lead to the introduction of a further symbol.

Hence, we finally obtain the semiclassical result for the determinant of $\mathbf{S}^{(1)}(ka)$

$$\det \mathbf{S}^{(1)}(ka) \sim e^{-i2\pi N(k)} \frac{\prod_{\ell=1}^{\infty} \left(1 - e^{-2i\pi\bar{\nu}_\ell(ka)}\right)^2}{\prod_{\ell=1}^{\infty} \left(1 - e^{2i\pi\nu_\ell(ka)}\right)^2} \,, \tag{42.1}$$

with the creeping exponential

$$\begin{aligned} \nu_\ell(ka) &= ka + e^{+i\pi/3}(ka/6)^{1/3} q_\ell + \cdots = ka + i\alpha_\ell(ka) + \cdots \quad (42.2) \\ \bar{\nu}_\ell(ka) &= ka + e^{-i\pi/3}(ka/6)^{1/3} q_\ell + \cdots = ka - i(\alpha_\ell(k^* a))^* + \cdots \\ &= (\nu_\ell(k^* a))^* \,, \quad (42.3) \end{aligned}$$

Remark 42.1 Sources. This section is based on appendix E of ref. [M.1].

There are three possibilities of "semiclassical" contact of the point particle with the disk j_i:

1. either geometrical which in turn splits into three alternatives
   (a) *specular reflection* to the right,
   (b) *specular reflection* to the left,
   (c) or *'ghost tunneling'* where the latter induce the nontrivial pruning rules (as discussed above)
2. or *right-handed creeping turns*
3. or *left-handed creeping turns*.

See figure **??**. The specular reflection to the right is linked to a left-handed creeping path with a knot. The specular reflection to the left matches a right-handed creeping path with a knot, whereas the creeping paths in the ghost tunneling case are topologically trivial. In fact, the topology of the creeping paths encodes the choice between the three alternatives for the geometrical contact of the disk. This is the case for the simple reason that creeping sections have to be positive definite in length: the creeping amplitude has to decrease during the creeping process, as tangential rays are constantly emitted. In mathematical terms, it means that the creeping angle has to be positive. Thus, the positivity of the *two* creeping angles for the left *and* right turn uniquely specifies which of the three alternatives, either specular reflection to the right or to the left or straight "ghost" tunneling through disk j, is realized for the semiclassical geometrical path. Hence, the existence of a saddlepoint is guaranteed.

References

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M. Berry recommends: Weyl formula for acoustics Balian and Bloch (1971)