Problem 1 (8.1):

$$\begin{aligned}\big(-\partial_x^2 + m^2\big)\Delta(x - x') &= \int \frac{d^4k}{(2\pi)^4}\frac{k^2 + m^2}{k^2 + m^2 - i\epsilon}e^{ik(x-x')} \\ &= \delta(x - x')\end{aligned}$$

Problem 2 (8.2):

$$\Delta(x - x') = \int \frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\int \frac{dk_0}{(2\pi)}\frac{e^{-ik_0(t-t')}}{-k_0^2 + \mathbf{k}^2 + m^2 - i\epsilon}$$

Integrate over k^0 with a contour integral in the complex plain. When $t > t'$ (vice $t < t'$), the contour is closed in the lower (vice upper) half-plane so that $\Re(-ik_0(t - t')) < 0$. Closing up includes a pole at $-\omega + i\epsilon$, for $\omega = \sqrt{m^2 + \mathbf{k}^2}$, and closing down gets the pole at $\omega - i\epsilon$ with a minus sign because the countour is clockwise. Computing the residues yields

$$\Delta(x - x') = i\int \widetilde{dk}\,\left(\theta(t - t')e^{ik(x-x')} + \theta(t' - t)e^{-ik(x-x')}\right)$$

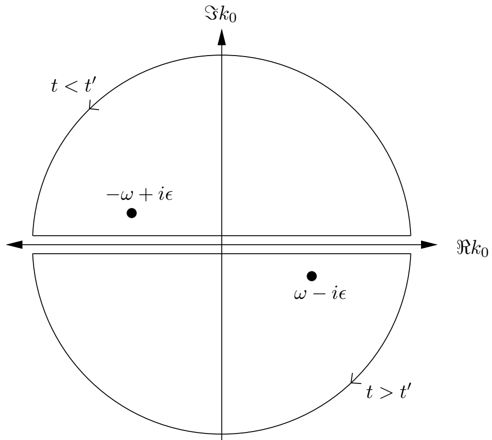

Figure 1: Contours and poles in the k_0 plane.

Problem 3:

Start with

$$\frac{\partial^2}{\partial t^2}\big(\theta(t-t')f(t)\big) = \theta(t-t')f''(t) + 2\delta(t-t')f'(t) + \delta'(t-t')f(t)$$

Since the $\delta(t-t')$ is well-defined only in the context of an integration over t or t', its derivative is defined in terms of integration by parts, so that

$$\delta'(t-t')f(t) = -\delta(t-t')f'(t)$$

and

$$\frac{\partial^2}{\partial t^2}\big(\theta(t-t')f(t)\big) = \theta(t-t')f''(t) + \delta(t-t')f'(t)$$

Using this, we have

$$\begin{aligned}\big(-\partial_x^2 + m^2\big)i\theta(t-t')\int \widetilde{dk}\, e^{ik(x-x')} &= i\theta(t-t')\int \widetilde{d}\big(-\omega^2 + \mathbf{k}^2 + m^2\big)e^{ik(x-x')} \\ &\quad + \delta(t-t')\int \widetilde{dk}\,\omega e^{ik(x-x')} \\ &= \frac{1}{2}\delta(t-t')\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}e^{-i\omega(t-t')} \\ &= \frac{1}{2}\delta(x-x')\end{aligned}$$

We got the last line by using $\omega^2 = \mathbf{k}^2 + m^2$ and noting that, because $\delta(t-t')$ vanishes unless $t = t'$, we can set $e^{-i\omega(t-t')}$ to 1. The second term in (195) yields an identical contribution.