Roger Penrose

THE ROAD TO REALITY

A Complete Guide to the Laws of the Universe





(a)

(c)



(b)

Fig. 8.2 (a) Constructing the Riemann surface for $(1 - z^3)^{1/2}$ from two sheets, with branch points of order 2 at 1, ω , ω^2 (and also ∞). (b) To see that the Riemann surface for $(1 - z^3)^{1/2}$ is topologically a torus, imagine the planes of (a) as two Riemann spheres with slits cut from ω to ω^2 and from 1 to ∞ , identified along matching arrows. These are topological cylinders glued correspondingly, giving a torus. (c) To construct a Riemann surface (or a manifold generally) we can glue together patches of coordinate space—here open portions of the complex plane. There must be (open-set) overlaps between patches (and when joined there must be no 'non-Hausdorff branching', as in the final case above; see Fig. 12.5b, §12.2).