Problem 2

Given the differential equation $\dot{x} + x + \epsilon x^2 = 0$ subject to the initial condition $x = 1$ at $t = 0$

a) Compute the approximate solution $x(t, \epsilon)$ using perturbation theory (assuming $|\epsilon| \ll 1$) up to terms of $O(\epsilon^3)$.

b) Compute the exact solution $x(t, \epsilon)$ using separation of variables.

c) Perform a series expansion of the exact solution for small ϵ and compare with the perturbation solution. Do your expansions agree?