Exercises

Exercise 26.1 **Period doubling in your pocket:** *Take a programmable pocket calculator or Matlab or whatever makes you feel good and program the function*

$$f(x) = \lambda - x^2 \,.$$

The game consists in staring at the display, and looking for regularities in the sequences of iterates.

(a) *(no thinking) Try to determine fixed point $x_* = f(x_*)$ by blind iteration. Chose some value of λ a bit bigger than 0, and initial x between -1 and 1. Enter the initial x_0 and read off the next x_1. Start again, with x_1 as input. The number x_2 appears on the display. Is it a fixed point? Press the button again, and again, until $x_n = x_*$ to desired accuracy.*

(f) *(no thinking) Increase λ in small steps, as long as the trajectory does not blow up, let transients die, and then plot few hundred consecutive x_n. Generate a figure to replace the hand drawn figure 26.14.*

(c) *(thinking) Determine the smallest positive λ for which almost any initial x_0 iterates to $-\infty$.*

(b) *(no thinking) Try $\lambda = 3/4$. How's the convergence now?*

(c) *(thinking) Determine λ for which the fixed point x_* goes unstable.*

(d) *(no thinking) Try also λ: 1, 1.31070274134, 1.38154748443, 1.3979453597.*

(e) *(thinking) Compute the next number in this series. Estimate Feigenbaum δ.*

(g) *(thinking) Determine numerically scaling factors α_m which overlay (approximately) neighborhood of $x = 0$ for superstable $f^{2(m-1)}(x)$ over the neighborhood for $\alpha_m f^{2^m}(f^{2^m}(x/\alpha_m))$ for 4, 8, 16, $\cdots$ superstable cycles. Draw a figure to replace the hand drawn figure 26.26.*