Exercise 11.7 **Unimodal map pruning.** *Consider the 1-d quadratic map*

$$f(x) = Ax(1-x)\,, \qquad A = 3.8\,. \tag{11.25}$$

(a) *(easy) Plot (11.25), and the first 4-8 (whatever looks better) iterates of the critical point $x_c = 1/2$.*

(b) *(hard) Draw corresponding intervals of the partition of the unit interval as levels of a Cantor set, as in the symbolic dynamics partition of figure 11.8(b). Note, however, that some of the intervals of figure 11.8(b) do not appear in this case - they are* pruned*.*

(c) *(medium) Produce ChaosBook.org quality figure 11.8(a).*

(d) *(easy) Check numerically that $K = S^+(x_c)$, the itinerary or the "kneading sequence" of the critical point is*

$$K = 1011011110110111101011110111110\ldots$$

The tent map point $\gamma(S^+)$ with future itinerary S^+ is given by converting the sequence of s_n's into a binary number by the algorithm (11.11),

$$\begin{aligned} w_{n+1} &= \begin{cases} w_n & \text{if } s_{n+1} = 0 \\ 1 - w_n & \text{if } s_{n+1} = 1 \end{cases}\,, \qquad w_1 = s_1 \\ \gamma(S^+) &= 0.w_1 w_2 w_3 \ldots = \sum_{n=1}^{\infty} w_n / 2^n\,. \end{aligned}$$

(e) *(medium) List the the corresponding kneading value (11.12) sequence $\kappa = \gamma(K)$ to the same number of digits as K.*

(f) *(hard) Plot the missing dike map, figure 11.10, in ChaosBook.org quality, with the same kneading sequence K as $f(x)$. The dike map is obtained by slicing off all $\gamma\left(S^+(x_0)\right) > \kappa$, from the complete tent map figure 11.8(a), see (11.13).*

How this kneading sequence is converted into a series of pruning rules is a dark art, relegated to sect. 13.6 and appendix E.1.