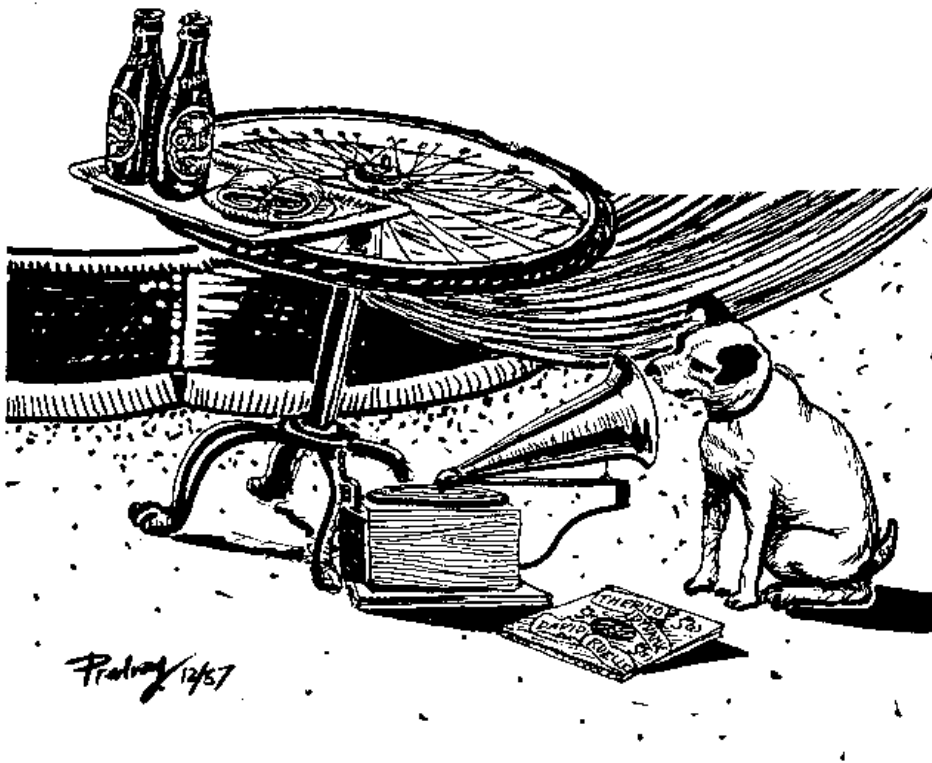


Chaos: Classical and Quantum

I: Deterministic Chaos



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No man but a blockhead ever wrote except for money
—Samuel Johnson

This book is a result of collaborative labors of many people over a span of several decades. Coauthors of a chapter or a section are indicated in the byline to the chapter/section title. If you are referring to a specific coauthored section rather than the entire book, cite it as (for example):

C. Chandre, F.K. Diakonov and P. Schmelcher, section “Discrete cyclist relaxation method,” in P. Cvitanović, R. Artuso, R. Mainieri, G. Tanner and G. Vattay, *Chaos: Classical and Quantum* (Niels Bohr Institute, Copenhagen 2010); ChaosBook.org/version13.

Do not cite chapters by their numbers, as those change from version to version. Chapters without a byline are written by Predrag Cvitanović. Friends whose contributions and ideas were invaluable to us but have not contributed written text to this book, are credited in the acknowledgments.

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Daniel Borrero Oct 23 2008, soluCycles.tex

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