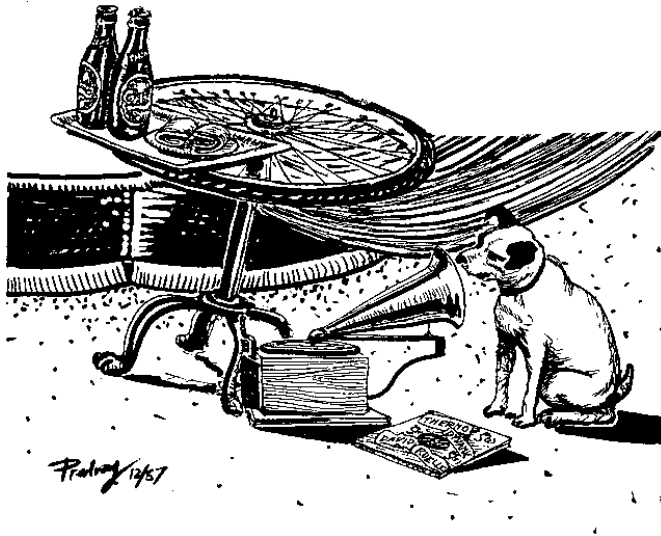


# Chaos: Classical and Quantum

## I: Deterministic Chaos



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## Contributors

No man but a blockhead ever wrote except for money  
—Samuel Johnson

This book is a result of collaborative labors of many people over a span of several decades. Coauthors of a chapter or a section are indicated in the byline to the chapter/section title. If you are referring to a specific coauthored section rather than the entire book, cite it as (for example):

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Rössler flow figures, tables, cycles in chapters 2, 13 and exercise 13.7

**Edward A. Spiegel**

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**Luz V. Vela-Arevalo**

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**Lei Zhang**

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