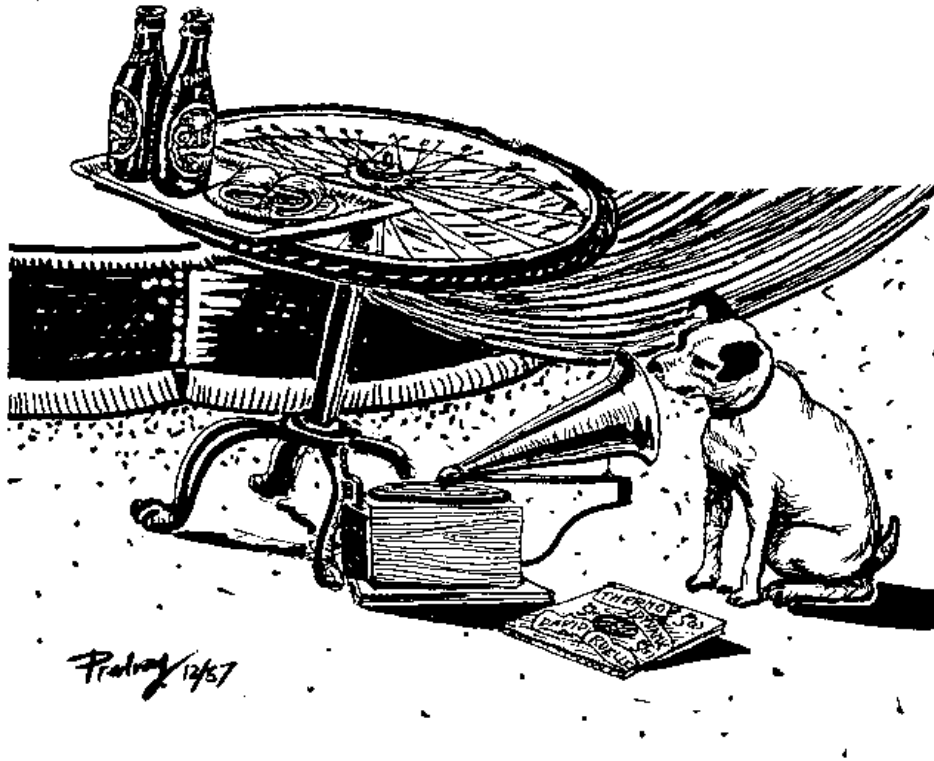


# Chaos: Classical and Quantum

## I: Deterministic Chaos



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## Contributors

No man but a blockhead ever wrote except for money  
—Samuel Johnson

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I can eke living on stories and little articles, that don't cost  
a tithe of the output a book costs. Why write novels any  
more!

—D.H. Lawrence

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