Appendix E

Counting itineraries

E.1 Counting curvatures

One consequence of the finiteness of topological polynomials is that the contributions to curvatures at every order are even in number, half with positive and half with negative sign. For instance, for complete binary labeling (18.7),

$$\begin{aligned} c_4 &= -t_{0001} - t_{0011} - t_{0111} - t_0 t_{01} t_1 \\ &\quad + t_0 t_{001} + t_0 t_{011} + t_{001} t_1 + t_{011} t_1 \,. \end{aligned} \tag{E.1}$$

We see that 2^3 terms contribute to c_4, and exactly half of them appear with a negative sign - hence if all binary strings are admissible, this term vanishes in the counting expression.

[exercise E.2]

Such counting rules arise from the identity

$$\prod_p \left(1 + t_p\right) = \prod_p \frac{1 - t_p{}^2}{1 - t_p} \,. \tag{E.2}$$

Substituting $t_p = z^{n_p}$ and using (13.15) we obtain for unrestricted symbol dynamics with N letters

$$\prod_p^{\infty} (1 + z^{n_p}) = \frac{1 - Nz^2}{1 - Nz} = 1 + Nz + \sum_{k=2}^{\infty} z^k \left(N^k - N^{k-1}\right)$$

The z^n coefficient in the above expansion is the number of terms contributing to c_n curvature, so we find that for a complete symbolic dynamics of N symbols and $n > 1$, the number of terms contributing to c_n is $(N-1)N^{k-1}$ (of which half carry a minus sign).

[exercise E.4]