In order to illuminate my problem, let us have a look at the 'prototype' 2-dimensional repeller, the single hyperbolic fixed point introduced in section 8.3, and try to calculate its escape rate:

$$f(z_1, z_2) = (\lambda_s z_1, \lambda_u z_2) \tag{1}$$

with $0 < \lambda_s < 1$ and $\lambda_u > 1$. Of course the Perron-Frobenius operator

$$\begin{aligned} \mathcal{L}h(z_1, z_2) &= \int_{\mathbf{R}^2} dw_1 dw_2 \, \delta((z_1, z_2) - f(w_1, w_2)) h(w_1, w_2) \\ &= \frac{1}{\lambda_s \lambda_u} h(z_1/\lambda_s, z_2/\lambda_u) \end{aligned} \tag{2}$$

has smooth eigenfunctions $\varphi_{n_1,n_2} = z_1^{-n_1-1} z_2^{n_2}$, so I should be happy.

The drawback is that I must choose a finite sized neighbourhood of the repeller in order to define an escape rate. For the example given here, let this region be $\mathcal{M} := [-1, 1] \times [-1, 1]$, and for the escape rate I have to examine

$$\Gamma_n = \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} dw \int_{\mathcal{M}} dz \, \delta(z - f^n(w)), \tag{3}$$

and this is

$$\Gamma_n = \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} dz \, \tilde{\mathcal{L}}^n i(z). \tag{4}$$

with $i(z) = 1$ and an operator

$$\tilde{\mathcal{L}}h(z_1, z_2) = \int_{\mathcal{M}} dw_1 dw_2 \, \delta((z_1, z_2) - f(w_1, w_2)) h(w_1, w_2) \tag{5}$$

which is definitely not the Perron-Frobenius operator defined in (2). The representation given by the second equality in (2) is valid for $\tilde{\mathcal{L}}$ only if $(z_1, z_2) \in f(\mathcal{M})$, and $\tilde{\mathcal{L}}h(z_1, z_2)$ is zero otherwise. The eigenfunctions of $\mathcal{L}$ cannot be the eigenfunctions of $\tilde{\mathcal{L}}$. So what has the escape rate to do with the Perron-Frobenius operator? Why does it work?

Another question: Which linear combination of the φ_{n_1,n_2}'s yields $i(z) = 1$? Smooth functions on $\mathcal{M}$ do not belong to the somewhat strange function space spanned up by the φ_{n_1,n_2}'s.