Understanding the Production of Photons from Vacuum Fluctuations Via the Dynamical Casimir Effect

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Recent developments in industry have pushed technology to the point where vacuum fluctuations matter at a practical level. Nanomachines and semiconductors are two such applications where there is a current need to understand the Casimir Effect and how to use it when possible and avoid it when necessary. This requires an understanding of what is happening at a fundamental level. To the observer naive to quantum field theory, there is no particular reason to think there should be a force in these limits. The following is a presentation of the background of the subject culminating with a discussion of the most recent observation of photons perturbed from the vacuum by a superconducting quantum interference device.

INTRODUCTION

In the course of study of quantum field theory we often talk of self-energy diagrams as vacuum to vacuum transitions. We start with some observable and let something happen to it, which we have no ability to observe, and then examine the result. Typically, we ignore the fact that there can be underlying disturbances to the vacuum state itself so that a vacuum transition may not necessarily have the same beginning and end in the presence of some random perturbation. Recent experiments by Wilson et al[1][2] have given us the opportunity to recognize that there is something real to the mathematical formulism of the Green's functions that represent all possibilities for a transition. It is possible to interrupt the vacuum to vacuum transition and observe photons as a result.

1. STANDING ON THE SHOULDERS OF GIANTS

Casimir

In the late 40's Casimir was interested in the London-Van der Waal's force between neutral atoms[3]. He proposed a solution to this interaction at large distance[4], which for a perfectly conducting plate with a neutral atom resulted in,

$$\delta E = \frac{-3\hbar c\alpha}{8\pi R^4}.$$

Whereas, for two particles interacting over large distance,

$$\delta E = \frac{-23\hbar c\alpha_1\alpha_2}{4\pi R^7}.$$

Casimir realized through some correspondence that these expressions, derived from taking Van der Waal's forces and correcting for retardation, could be obtained by looking at the zero point energy of the system and calculating the change in this quantity.

He proposed to look at a cubic volume with side of length L1, where each side is a perfect conductor. Inside this system he considered two configurations. One where there was a perfect conductor inserted some tiny distance a from one side, displacing along the z-axis, and the other with the same conductor inserted at $\frac{L}{2}$ With this scenario we know that there are the following relations,

$$k_x = \frac{\pi n_x}{L}, k_y = \frac{\pi n_y}{L}, k_z = \frac{\pi n_z}{a}$$

and

$$|k| = \sqrt{k_x^2 + k_y^2 + k_z^2}.$$

Now we consider that inside the cavity the minimum energy, zero point energy, is defined by the sum,

$$E = \frac{1}{2}\sum_i \hbar\omega_i$$

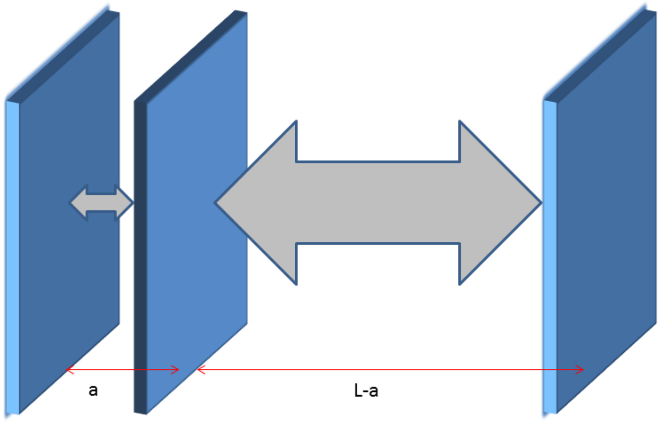

FIG. 1: Casimir proposed a cavity with three perfectly conducting mirrors positioned at 0, a and L.

where the index represents a sum over the resonant frequencies of the cavity.

We cannot make any interpretation by this alone because this sum is divergent; however, we can make some interpretation on modes that are within the small space with respect to modes in the larger space.

Now we can convert this sum to an integral in polar coordinates,

$$\frac{1}{2}\sum_i \hbar\omega_i = \frac{2\hbar c L^2}{2\pi}\sum_{(0)1}^{\infty}\int_0^{\infty} d\rho\rho\sqrt{\frac{n^2\pi^2}{a^2}+\rho^2}$$

The difference in the two energies is

$$\delta E = \frac{2\hbar c L^2}{2\pi}\{\sum_{(0)1}^{\infty}\int_0^{\infty} d\rho\rho\sqrt{\frac{n^2\pi^2}{a^2}+\rho^2} - \int_0^{\infty}\int_0^{\infty} d\rho dk_z\frac{a\rho}{\pi}\sqrt{k_z^2+\rho^2}\}$$

We note that the notation on the summation implies that there is a ground level quantity not represented by the integral that is included in the summation. This integral is still divergent because there are an infinite amount of modes that can fit inside the cavity. To regularize this integral Casimir multiplied the integrand by a function that went to zero rapidly when the ratio of wave numbers $(\frac{k}{k_m}) \to \infty$ and approached unity as $k \ll k_m$. This trick and the application of the Euler-Maclaurin formula led to obtaining

$$\frac{\delta E}{L^2} = \frac{-\hbar c\pi^2}{720a^3}$$

and a force

$$F = \frac{\hbar c\pi^2}{240a^4}.$$

The function that allows us to obtain these results can be intuited as a measure of what type of waves can see this plate. In the case where the wavelength is small compared to the atomic spacing, the neutral perfect conducting place is not even relevant to the wave an example would be x-rays.

Lifshitz

Following in Casimir's footsteps, Lifshitz extended the Casimir effect to include dielectric media[5]. In this case it is necessary to take a macroscopic approach and determine the force of this interaction by the electromagnetic fields that fluctuate within the bodies and vacuum. He began by calculating the E and H fields for the interior of each dielectric body and for the vacuum separating them. The equations for these monochromatic fields in a dielectric nonmagnetic medium are

$$\nabla \times E = i\frac{\omega}{c}H$$

$$\nabla \times H = -i\frac{\omega}{c}(sE + K)$$

Here K represents a field included to represent the random nature of the fluctuations of the fields. For the sake of space, much of this calculation will be omitted and I will skip to the resulting force,

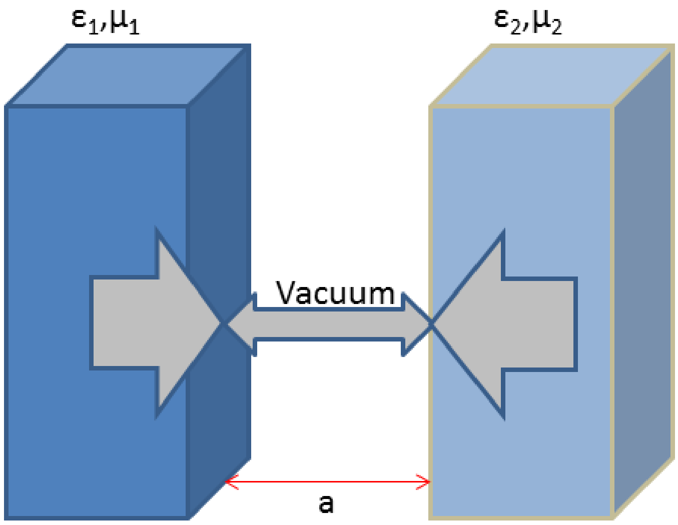

FIG. 2: Lifshitz investigated the scenario where there were two dielectrics with different permeabilities that were separated by vacuum with a being the size of separation.

$$f = \frac{\hbar}{2\pi^2 c^3} Re \int_0^\infty \int dp d\omega p^2 \omega^3 \coth\frac{\hbar\omega}{2T}\{(\frac{(s_1+p)(s_2+p)}{(s_1-p)(s_2-p)}e^{-2ip\omega l/c} - 1)^{-1} + (\frac{(s_1+\varepsilon_1 p)(s_2+\varepsilon_2 p)}{(s_1-\varepsilon_1 p)(s_2-\varepsilon_2 p)}e^{-2ip\omega l/c} - 1)^{-1}\}$$

The terms s_1 and s_2 are defined as,

$$s_1 = \sqrt{\varepsilon_1(\omega) - 1 + p^2}$$

$$s_2 = \sqrt{\varepsilon_2(\omega) - 1 + p^2}$$

Lifshitz makes extensive discussion of the method to integrate this by contour integration and his choice of contour. He then examines the limits where the wavelength is either small or large when compared to the absorption spectrum of the dielectric bodies. In the case of small separation he finds that for two identical dielectrics $\varepsilon_{10} = \varepsilon_{20} = \varepsilon_0$ with ξ representing imaginary ω, $\xi = -i\omega$,

$$F = \frac{\hbar}{8\pi^2 l^4}\int_0^\infty d\xi (\frac{\varepsilon(i\xi)+1}{\varepsilon(i\xi)-1})^2$$

When a large separation is considered, the result originally obtained by Casimir is found. Here we look at two dielectrics,

$$F = \frac{\hbar c \pi^2}{240 l^4}(\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1})^2 \phi(\varepsilon_0)$$

where

$$\phi(\varepsilon_0) = 1 - \frac{1.11}{\sqrt{\varepsilon_0}} ln \frac{\varepsilon_0}{7.6}$$

Considering the situation where there is a dielectric and a perfect conductor ($\varepsilon_{20} = \varepsilon_0 and \varepsilon_{10} \rightarrow \infty$) the power of the $\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}$ term reduces to 1. Then taking the case where the materials are sufficiently rarified (gases), the earlier integral is expanded and reduces to

$$F = \frac{23\hbar c}{640\pi^2 l^4}(\varepsilon_{10} - 1)(\varepsilon_{20} - 1)$$

The energy that corresponds with this force corresponds to the energy that Casimir found, which here is found via examining macroscopic quantities, whereas, Casimir was investigating the interaction of a single atom with another atom.

Lifshitz made one additional contribution beyond this, where he incorporated a temperature dependence into this calculation. I will not repeat it here because it was incorrect, but a necessary observation nonetheless.

A more accurate derivation of the temperature dependence of this effect was separately but simultaneously achieved by Mehra[6], and several other groups that concur that Lifshitz made an error. The following are Mehra's two corrections to the force,

$$F_1 = \frac{\pi^2 \hbar c T'}{L^3} \sum_{n=0}^{\infty} \ln(1 - e^{-n/LT'})$$

and

$$F_2 = \frac{\pi^6 \hbar c T'^4}{45}$$

These terms generally don't match what Lifshitz derived although at high temperatures the $1/L^3$ character of the interaction is the same.

Schwinger

In the context of what has been covered in the quantum field theory course, Schwinger's derivation of the Casimir effect[7] is most similar to how the material has been covered.

We can say that

$$< 0_+ | 0_- >= e^{iW(J)}$$

where

$$W(J) = \frac{1}{2} \int [dx][dx'] J(x) D(x - x') J(x').$$

Here we have defined $J(x^i)$ as a source and $D(x - x')$ is the propagator that takes the state from x to x'. We can write $D(x - x')$ as

$$D(x - x') = \int \frac{dp}{(2\pi)^3} e^{ip\cdot(r - r')} \frac{i}{2E} e^{-iE|t - t'|}$$

The source can be found from the field by taking

$$J(x) = \int [dx'] D^{-1}(x - x') A(x').$$

We can write D^{-1} as

$$D^{-1}(x - x') = \int \frac{[dp]}{(2\pi)^3} e^{ip\cdot(r - r')} \left\{ E^2 + \frac{\partial^2}{\partial t^2} \right\} \delta(t - t').$$

When the boundaries of the conducting surfaces are allowed to be added and removed as in the original setup by Casimir, there is no longer spatial translational invariance in the propagator so that from here we must write $D(x-x')$ as $D(x,x')$. By changing the parameters of the configuration of the conducting walls a change in $W(J)$ is induced.

$$\delta W(J) = \frac{1}{2}\int [dx][dx']J(x)\delta D(x,x')J(x') = -\frac{1}{2}\int [dx][dx']A(x)\delta D^{-1}(x,x')A(x')$$

To establish what this new term is we can compare $i\delta W$ to the two photon process as described by

$$\frac{1}{2}\{i\int [dx]J(x)A(x)\}^2$$

where

$$iJ(x)J(x')|_{eff} = -\delta D(x,x').$$

Using this gives us a different expression for δW

$$\delta W_0 = \frac{i}{2}\int [dx][dx']D(x,x')\delta D^{-1}(x,x').$$

When we make time explicit $t-t' \to \tau$ and observe the factor $e^{-i\delta\epsilon T}$ with T being the time interval during which the conducting plate is introduced and $\delta\epsilon$ being an energy shift represented in our previous integral.

$$\delta\epsilon = -\frac{i}{2}\int [dr][dr']d\tau D(r,r',\tau)\delta D^{-1}(r,r',\tau)$$

Now we define a series of conducting plates in the same manner as Casimir. There are conducting plates at $z=0,a,L$. There is also one at $-L$ in Schwinger's derivation, but it serves no function as he says himself.

There is a set of normalized eigenfunctions, $\mu_n(z)$ with eigenvalues E,

$$\mu_n(z) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi z}{a}), E^2 = p_\perp^2 + (\frac{n\pi z}{a})^2$$

So that we can write the propagator as

$$D_a(x,x') = \int \frac{[dp_\perp]}{(2\pi)^2}e^{ip_\perp\cdot(r-r')}\sum_n \mu_n(z)\mu_n(z')\frac{i}{2E}e^{-iE|t-t'|}$$

This allows the energy perturbation to be written

$$\delta\epsilon_a \ \frac{A}{2}\int \frac{dp\perp}{(2\pi)^2}\sum_n \delta E^2\frac{1}{2E}e^{-iE\tau}|_{\tau\to 0'}$$

Following some manipulation Schwinger obtains

$$\frac{\delta\epsilon_{L-a}}{A} = -\frac{1}{4\pi}\delta a\frac{1}{i\tau}\frac{d^2}{d\tau^2}\frac{1}{i\pi\tau},$$

which he expands to obtain

$$F = -\frac{\pi^2}{480a^4}$$

This result matches the known electromagnetic result when the polarization is accounted for by multiplying by 2.

Schwinger also incorporated a result for the temperature dependence, which he reported as

$$F_T = -\frac{\zeta(3)kT}{8\pi a^3}$$

Here again we multiply by two to account for polarization to obtain the known electromagnetic result.

2. THE TREES THAT ARE SLOW TO GROW BEAR THE BEST FRUIT

Measuring the force determined by Casimir is a non-trivial exercise. It has been an ongoing venture for more than 60 years spanning various types of surfaces and geometries. Many experiments rely on the geometry where there is a sphere with a large radius that is near to a plate. This radius is large enough so that it looks almost flat. Another interesting point about geometry was numerically solved by Boyer in 1968, where he found that spherical half-shells actually repel each other rather than attract[8]. This underlines the necessity of correcting for different geometrical configurations. Technical limitations in the ability to position and measure different parameters have abounded and only gradually been overcome. The following will provide a brief account of some of the more important experiments.

10 years after Casimir's paper

In 1957 Derjaguin et al[9] measured the attraction between plates of fused quartz3 at separations of $0.1-1\times10^{-4}$cm. Derjaguin used a force that can be represented as

$$f(h) = 2\pi^2\rho\frac{3\hbar e^4}{m^2h^6}\int_0^\infty\int_0^\infty d\omega_1 d\omega_2\frac{\phi(\omega_2)\phi(\omega_1)}{\omega_1\omega_2(\omega_1+\omega_2)}$$

In this equation h is the separation between the surfaces and ρ is the radius of the spherical surface. Derjaguin found it quite challenging to keep the surface clean and free of electric charge from the cleaning itself.

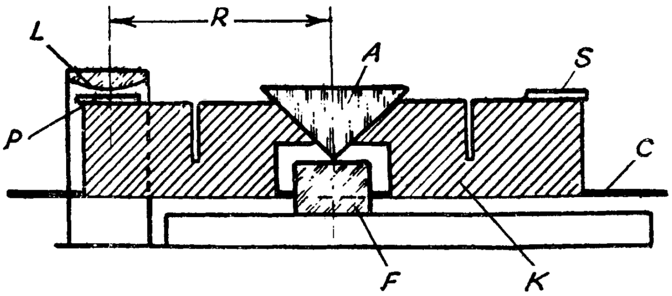

FIG. 3: Derjaguin investigated the Casimir force with his collaborators Brikosova and Lifshitz using a beam balance with a quartz lens and a quartz plate as shown above. Here L is the spherical lens. P is the plate it interacts with. A is a prism that serves as a pivot and F is the fulcrum it rests on. S is a mirror and C is a counterweight.

Likewise Sparnaay measured the attraction of metal plates[10]. Sparnaay used a device with a spring and a counterweight that held a flat plate above another flat plate and measured the capacity of a condenser to determine deflection.4 In both of these experiments they determined forces that were in agreement with the Casimir effect considering their experimental error.

Lifshitz calculations were confirmed in 1973 by Sabisky and Anderson [11] in an experiment where alkaline earth fluoride SrF_2 crystals were covered with a film of helium superfluid and the force holding the thin film to the substrate was measured. This was in very good agreement with Lifshitz theory.

Almost 50 years later

In the late 80's and during the 90's, advances in experiments to measure the Casimir force occurred. In 1992, Sukenik et al[12] created an apparatus that included two gold mirrors5 that were angled together to form a v. These mirrors deflected the ground state energy of a beam of incoming sodium atoms and a laser excited atoms at a certain height leaving the cavity, whereupon they are detected by field ionization. After considering several possible sources of this shift, Sukenik found that the QED model proposed by Casimir was the correct model. In 1997 Lamoreaux[13] used a torsion pendulum sandwiched between two condensers to detect the Casimir force. The device measured the capacitance of the condenser on each side as a plate was brought near to one end of the pendulum. Lamoreaux was able to show in a limited range that there was a Casimir force to 5% accuracy.

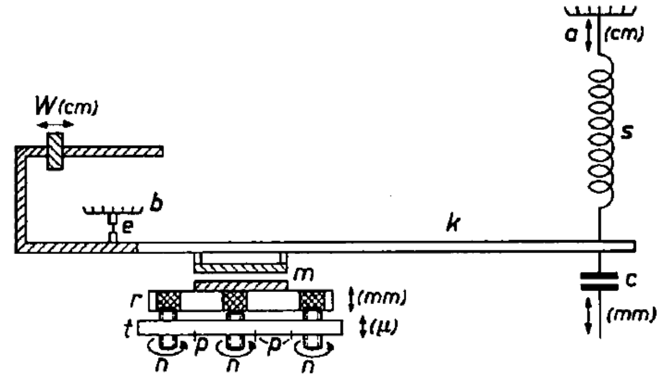

FIG. 4: Sparnaay developed an apparatus that counterbalanced a plate over another plate using a spring and a wire attached to fixed mounts. The capacitance was then measured between two plates mounted below the spring. Alignment of parallel surfaces was intricate and done using several micrometers.

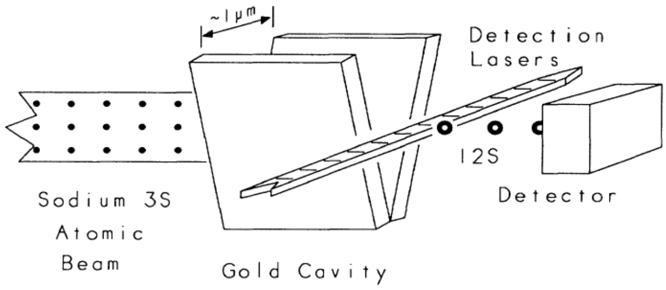

FIG. 5: Sukenik constructed a device which passed sodium ions through a channel between two gold plates. The ions at a height were excited upon exiting and impinged on a device that would measure them.

Mohideen et al took another approach at detecting the Casimir force in 1998[14]. He used an apparatus7 that compared the signal of two photodiodes receiving laser light that has been reflected from a cantilever attached to a metal sphere which is brought in proximity with a metal surface.

Past the year 2000

Several more recent experiments have been performed to measure certain aspects of the Casimir force. In 2002 Bressi et al measured the Casimir effect across a much broader range, $0.5 - 3\mu m$ with 15% precision[15]. Bressi used a plate mounted to a piezo electric and a cantilever and measured the capacitance between the two. The most noteworthy portion of his experiment is the expanded range and resolution of $50nm$ step spacing.

The next interesting advancement of experiments regarding the Casimir effect is the observation by Lisanti et al of the effect of varying the skin-depth of the metallic surface[16]. This was accomplished by coating a polystyrene sphere with films of different depths and different materials. They determined that skin-depth had a non-negligible effect of reducing the force systematically by 17%. This was less in agreement with theory than a thick metallic surface.

3. BUT, SOFT! WHAT LIGHT THROUGH YONDER WINDOW BREAKS?

Dynamical Casimir Effect

In 1976 Fulling et al predicted that a mirror accelerating would produce radiation from the vacuum[17]. In their terms, the description of a mirror in the presence of an electromagnetic field in $2D$ spacetime would require boundary conditions such that a wave packet would perfectly reflect at the boundary. Therefore, at the beginning and end

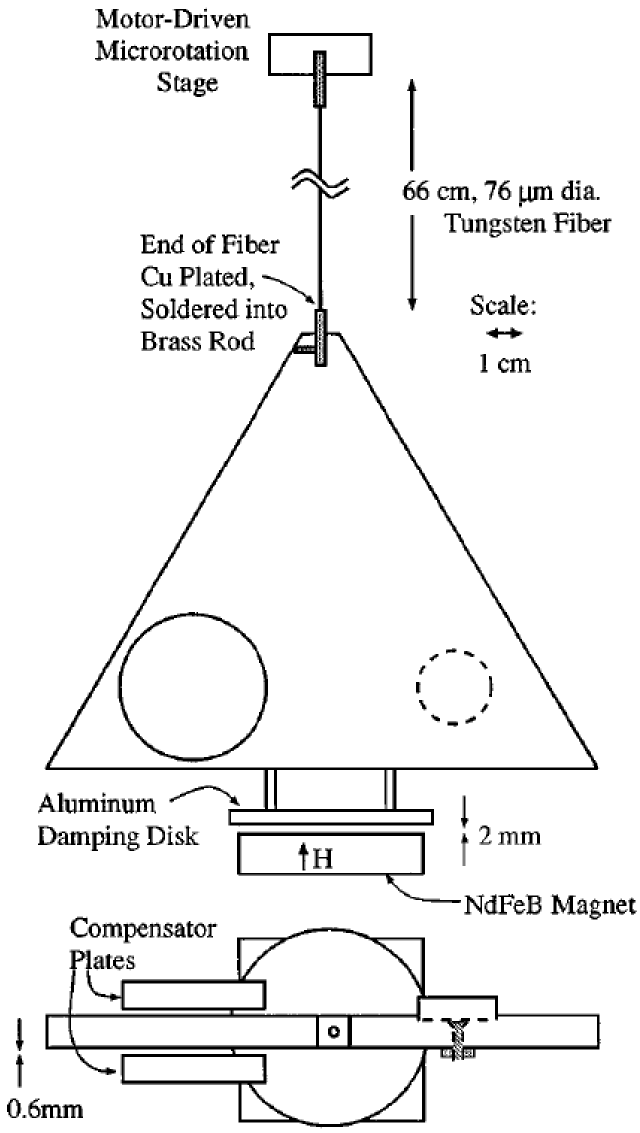

FIG. 6: Lamoreaux constructed a pendulum, which would be brought into proximity of a plate at one end. Force was measured by the capacitance between the pendulum and two plates on either side of it. The range was dependent of a feedback system, which operated a series of piezoelectric devices designed to keep the capacitance the same on both sides.

of the acceleration where the mirror is stationary 'particle language applies' and we can predict the creation of real particles, radiation.

Fulling defined $\phi(x,t)$ to be a field and then used the operator form and expanded it into eigenfunctions,

$$\phi(x,t) = \int_0^\infty d\omega[a_\omega^{in}\phi_\omega + a_\omega^{in\dagger}\phi_\omega^*]$$

Now we look at the energy momentum tensor $T_{\mu\nu}(t,x)$ where it is defined as a sum of a normal ordered component and its expectation value in the vacuum state,

$$< T_{\mu\nu} >= \int_0^\infty d\omega T_{\mu\nu}(\phi_\omega, \phi_\omega^*)$$

Where Fulling specifies that this integrand is evaluated as a bilinear form on the mode functions ϕ and ϕ^*. Next he inserts a ϵ into time for the complex conjugate so that they are not evaluated at exactly the same point in spacetime.

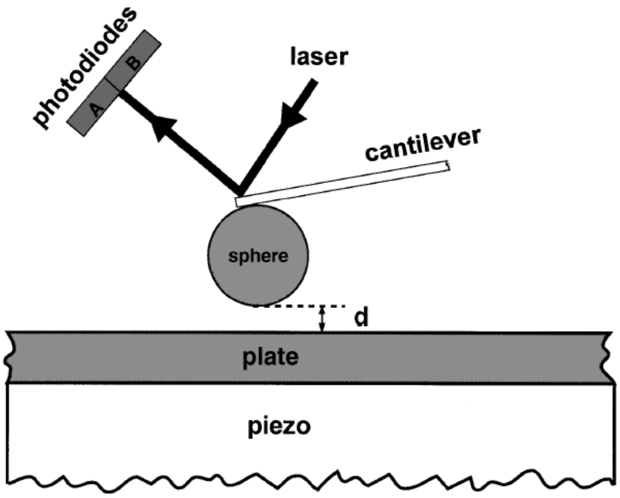

FIG. 7: Here we can see the bead attached to an AFM tip as a cantilever. The laser impinges on the cantilever surface and reflects to a pair of photodiodes whose intensities are compared to determine the amount of deflection of the cantilever.

In the following $u = t - x$ and $v = t + x$ and $'$ represents a derivative.

$$\frac{\partial\phi}{\partial t}, \frac{\partial\phi}{\partial x} = \sqrt{\frac{\omega}{4\pi}}[e^{-i\omega v} \mp p'(u)e^{-i\omega p(u)}]$$

and

$$\frac{\partial\phi^*}{\partial t}, \frac{\partial\phi^*}{\partial x} = \sqrt{\frac{\omega}{4\pi}}[e^{-i\omega(v+\epsilon)} \mp p'(u+\epsilon)e^{-i\omega p(u+\epsilon)}]$$

So that the terms of the EM tensor are,

$$< T_{00} >=< T_{11} >, < T_{10} >=< T_{01} >= \frac{1}{4\pi}\int_0^\infty \omega d\omega[e^{i\omega\epsilon} \pm p'(u)p'(u+\epsilon)e^{i\omega[p(u+\epsilon)-p(u)]}]$$

Evaluating the integral and expanding in powers of ϵ leads to

$$< T_{00} >= -\frac{1}{2\pi\epsilon^2} - < T_{10} >$$

and considering when evaluating the derivatives of p at u

$$< T_{01} >= -\frac{\sqrt{p'}}{12\pi[\sqrt{p'}]''} + O(\epsilon)$$

The last step is to take the $\epsilon \to 0$ limit and discard the divergent $< T_{00} >$ leading term.

$$< T_{00}(u) >= - < T_{10}(u) >= \frac{\sqrt{p'}}{12\pi[\sqrt{p'}]''}$$

This characterization of the energy momentum tensor tells us about the radiation by the mirror that represents the existence of particles, which were created by the perturbation of the vacuum by the accelerating mirror. Following this Fulling investigates a series of cases of this effect including the case where there are two mirrors rather than just one.

Temperature dependence was investigated for this effect by Plunien et al[18]. They determined that the dependence of the number of photons produced by the dynamical Casimir effect with respect to temperature could be represented as,

$$< \tilde{N}_A >\approx< \tilde{N} >_0 -\delta_A \sinh^2(\frac{\varepsilon\Omega_0^1 T}{2})(1 + 2 < \tilde{N}_1 >_0)$$

Therefore, if one could find a state where the vacuum fluctuations could be controlled while temperature was finite but non-zero, then the photon production could be considerably enhanced.

Another important step in observing this effect was the proposition by Braggio et al to use a virtual mirror rather than a mechanical mirror[19]. Their idea involved circumventing the necessity of great amounts of power to sustain a mechanical oscillator. Braggio proposed to excite a plasma within a semiconductor in order to generate the effective motion of a mirror at relativistic speeds. They proposed using this as one wall in an electromagnetic resonant cavity. They investigated the effectiveness of using a niobium cavity with a copper mirror at one end that had a wafer of semiconductor layered on top of it. They also kept the cavity at $4.6K$.

In 2009 Johansson et al [20] published an article about a device that incorporated a super-conducting quantum interference device, QUID, to be able to perturb the vacuum using a single accelerating mirror. The SQUID behaves as a mirror to electromagnetic radiation that impinges on it from a coplanar waveguide8. The position of the mirror is controlled by the amount of magnetic flux that passes through the squid. This flux is provided by a driveline that generates a field which passes through the SQUID in a perpendicular direction. In 2013, Lähteenmäki et al[21] were able to accomplish something similar with a SQUID.

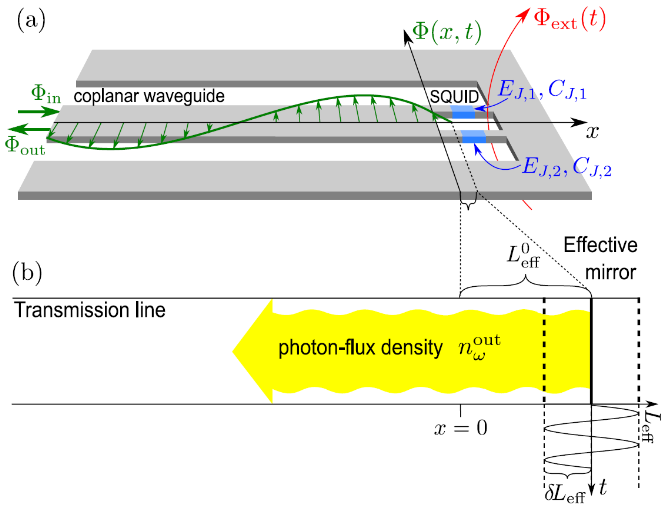

FIG. 8: Here we see the coplanar waveguide that terminates with a SQUID to have a variable boundary. The size of the boundary is controlled by $\phi_e xt(t)$. By measuring the photon flux density they were able to determine that photons were being excited by vacuum fluctuations.

To look at some of the math that lies behind this we consider the second quantization approach to this problem where we have some operator for the field $\phi(x)$, which we make time dependent by,

$$\phi(x,t) = e^{iHt}\phi(x)e^{-iHt}$$

Then we check the time dependence from the Heisenberg equation of motion which gives,

$$\frac{\partial^2}{\partial t^2}\phi = (\nabla^2 - m^2)\phi$$

We know that the field operator would be convenient to define as follows and that it will obey the massless Klein-Gordon equation in one dimension inside the transmission line.

$$\phi(x,t) = \int^t dt' E(x,t')$$

which becomes

$$\phi_{in/out} = \sqrt{\frac{\hbar Z_0}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} (a_{in/out}(\omega) e^{-i(\mp k_\omega x + \omega t)} + a^\dagger_{in/out}(\omega) e^{i(\mp k_\omega x + \omega t)})$$

Here $k_\omega = \omega/c_0$ is the wavenumber of the radiation produced by the mirror and the speed of light is based on the capacitance and inductance of the transmission line $c_0 = 1/\sqrt{L_0 C_0}$. We can also specify the inductance of the SQUID

$L_j(\phi_{ext}) = (\phi_0/2\pi)^2 E_j(\phi_{ext})$. The static magnetic flux condition allows the annihilation operator to be written as $a_{out}(\omega) = R(\omega)a_{in}(\omega)$. $R(\omega)$ is the reflection coefficient and can be written $R(\omega) = -e^{2ik_\omega l_e}$. Where, l_e is the length of the transmission line $l_e(\phi_{ext} = L_j(\phi_{ext})/L_0$. However, to obtain a photon flux density we need to have non-uniform acceleration so that we modify the annihilation operator to,

$$a_{out}(\omega) = R(\omega)a_{in}(\omega) + S(\omega)a^{\dagger}_{in}(\omega_d - \omega)$$

Where ω_d is the sinusoidal drive frequency and $S(\omega)$ is defined based on drive amplitude δl_e and spectrum amplitude $A(\omega)$ as follows.

$$S(\omega) = -i\frac{\delta l_e}{c_o}\sqrt{\omega(\omega_d - \omega)}A(\omega)A^*(\omega)$$

Finally, we can write down the number density of photons excited in the typical way and then write it in terms of $|S(\omega)|^2$.

$$n_{out}(\omega) =< a^{\dagger}_{out}(\omega)a_{out}(\omega) >= n_{in}(\omega) + |S(\omega)|^2 n_{in}(\omega_d - \omega) + |S(\omega)|^2$$

The first two terms according to Wilson are 'purely classical effects' that represent the reflection and upconversion of the input field to the drive frequency. The last term represents the photons excited from the dynamical Casimir effect.

If one is interested in the number of photons per second from some bandwidth $\Delta\omega$ then the expression is given by

$$N = \frac{1}{2\pi}\int_{\Delta\omega} d\omega n^{out}_{\omega} \approx \frac{\Delta\omega}{2\pi} n^{out}_{\omega}$$

Auch der Kleinste Feind ist Nicht zu Verachten.

To confront societies growing demand for smaller products, engineers have developed devices that are push past the limit of space in classical constraints and have entered the realm of the quantum. Electronics are being developed that use the Casimir effect for a type of FET transistor and simple nano machines are being developed that will allow devices to function even in the presence of interfering electromagnetic fields.

People who are developing micro electromechanical machines and nano electromechanical machines are very interested in the phenomenon of stiction. In order for these devices to work properly, they must not bind or jam. Serry et al[22] have determined that in these tiny devices the Casimir force begins to manifest itself and causes these devices to fail. Serry has also proposed that the Casimir force can be used in sensors in the form of an anharmonic oscillator.

There are devices that actually take advantage of the Casimir force. One such device created by Chan et al[23] uses the a metallic sphere and a plate to create a torsion device that can turn the plate about two rods. This occurs due to the attraction between the plate and the sphere. Another common structure that people are working with to study the applications of the Casimir force to MEMS and NEMS is a set of interlacing combs. These structures use the comb teeth to look at an array of parallel surfaces that can interact in this manner[24].

One example of using engineering to manipulate the geometry of the Casimir effect is the possibility to take a left-handed metamaterial and use it as a lens to reverse the direction of the Casimir force felt by a surface and produce levitation[25]. The idea of Leonhardt et al is that the Casimir force will reverse if you sandwich a perfect lens between two conducting plates.

The Casimir effect can be strongly suppressed by design due to its dependence on geometry, as shown by Boyer. An example of a structured surface that is engineered to reduce the force felt is given by Intravaia et al[26]. They created a deep grooved surface with structures that are less than the plasma wavelength, which in this case was features that were sub $100nm$. This lead to a suppression of the Casimir force beyond any suppression expected by theoretical calculation or achieved prior.

Ubi Dubium Ibi Libertas

The Casimir effect has been fertile ground for many different ideas ranging from the cosmological constant to sonoluminesence. It is interesting that an effect so relevant at a microscopic scale can be translated into a candidate to explain the cosmological constant. We can see that the vacuum energy density $\mathcal{E}$ from the stress-energy tensor $T_{\mu\nu}$ appears in Einstein's equations[27].

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E}g_{\mu\nu})$$

If we account for the vacuum energy density on the right hand side of the equation then $\tilde{T}_{\mu\nu}$ is the contribution of the tensor above the vacuum fluctuations. Now this term is equivalent to the cosmological constant that describes the inflation of the universe that is typically added on the left hand side e.g. $\lambda = 8\pi G\mathcal{E}$. Jaffe argues that our ability to formulate the Casimir effect in a multitude of ways suggests that there is no particular evidence that the vacuum energy itself is a real quantity. Jaffe cites Schwinger's QED formulation of the effect as one reason why we cannot assume a priori that vacuum energy is real. This is shown by Casimir's own statement that we cannot measure the vacuum energy, just the difference between vacuum energy inside and outside of two closely proximate plates. Jaffe cites Schwinger's QED formulation of the effect as one reason why we can't assume a priori that vacuum energy is real. Another interesting argument regarding the cosmological constant question is that if the vacuum energy difference was selected by a relevant length scale for cutoff there would be vastly stronger coupling of this effect to the expansion of the universe according to the calculations of Mahajan et al[28].

Another process that is not well understood and had people such as Schwinger, beginning with this paper[29], interested in applying the Casimir effect is Sonoluminesence, where a tiny bubble inside water will undergo repeated collapse and expansion. Each collapse corresponds with the emission of millions of photons due to the conversion of acoustic energy into photons[30]. This effect can occur tens of thousands of times per second and has been observed to persist for months. Schwinger proposed comparing the vacuum energy of the bulk water to the vacuum energy of bubble in order and he found,

$$E_{cavity} = 2\frac{4\pi}{3}R^3\int_0^K \frac{4\pi k^2 dk}{(2\pi)^3}\frac{\hbar ck}{2}(\frac{1}{\sqrt{\epsilon_{inside}}} - \frac{1}{\sqrt{\epsilon_{outside}}}) + \cdots$$

He termed this effect the dynamical Casimir effect, however, his effect is patently different from the one described above. He compared the energy of the expanded bubble to the collapsed bubble, which is sort of a quasistatic process rather than a dynamic process. Milton and Ng go into detail about reasons why they believe Schwinger's approach is flawed and does not accurately represent the physics[31]. They disagree with the reasoning behind the renormalization and argue that the bulk energy, $E = 1/2\Sigma\hbar\omega$ cannot be relevant. They make an adiabatic approximation that the remaining finite energy inside the volume scales as,

$$E_{Casimir} \approx \frac{(n-1)^2}{64a}$$

CONCLUSION

Many different ideas have come due to the concept of both the Casimir force and a zero point energy. We have just begun to probe the possibilities that the Casimir force provides with respect to microchip design. More intriguing are the various controversial ideas that people have put forth citing the Casimir effect as their basis, such as, explanations of the cosmological constant and sonoluminesence. There is even disagreement whether the Casimir effect is evidence of a zero point energy.

It seems that soon quantum effects like this are going to begin to dominate manufacturing and become relevant on a very significant level. The groundwork for this effect has been explored from a somewhat naive yet intuitive formulation of two perfectly conducting closely separated plates to reformulation in QED. The Casimir effect's dependence on geometry leaves the door open for explanations of both repulsive and attractive behaviors. The effect is far from fully analyzed and characterized and there is much still left unknown or at least uncertain in what role it takes in these processes.

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