Kuramoto-Sivashinsky weak turbulence, in the symmetry unrestricted space

by
Huaming Li
School of Physics, Georgia Institute of Technology, Atlanta, 30318
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Abstract

I. Introduction

Kuramoto-Sivashinsky equation was introduced by Kuramoto [1976] in one-spatial dimension, for the study of phase turbulance in the Belousov-Zhabotinsky reaction. Sivashinsky derived it independently in the context of small thermal diffusive instabilities for laminar flame fronts. It and related equations have also been used to model directional solidification and , in multiple spatial dimensions, weak fluid turbulence[1].[2]

In recent years unstable periodic orbits have been shown to be an effective tool in the nonlinear dynamical systems[3].[4]The theory has been successfully applied to low-dimensional ordinary differential equations (deterministic chaos) and linear partial differential equations (semiclassical quantization). Since[5],[6] it has been demonstrated that K-S Equations are rigorously equivalent to a finite dimensional dynamical system of ordinary differential equations(ODE's). In ref.,[7] it was shown that the periodic orbit theory can be used to describe spatially extended systems, by applying it as an example to the Kuramoto-Sivashinsky equation:

$$u_t = (u^2)_x - \upsilon u_{xx} - u_{xxxx}. \tag{1}$$

They applied Galerkin projection onto a subspace spanned by fourier modes with periodic boundary conditions,$u(x,t) = u(x + 2\pi, t)$. They isolated a smaller subspace of the system,$b_k = ia_k$, where a_k are real. By picking this subspace of the antisymmetric solutions $u(x,t) = -u(x,t)$, they eliminated the continuous translational symmetry. For that simplified system, spatiotemporally chaotic dynamics is described by means of an infinite hierarchy of its unstable spatiotemporally periodic solutions. An intrinsic parameterization of the corresponding invariant set serves as an accurate guide to the high-dimensional dynamics, and the periodic orbit theory yields several global averages characterizing the chaotic dynamics.

II Kuramoto-Sivashinsky system

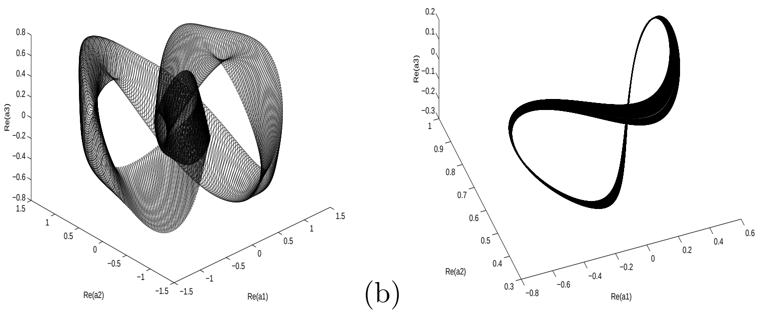


Figure 1: (a)My System in the quasiperiodic regime (greasy parameter $\upsilon = 0.04494382$, $N = 16$ modes truncation) (b)My System in the chaotic regime (greasy parameter $\upsilon = 0.121205$, $N = 8$ modes truncation)

Kuramoto-Sivashinsky system is one of the popular models to analyze weak turbulence or 'spatiotemporal chaos'. We shall study unstable spatiotemporallyperiodic solutions of the full Kuramoto-Sivashinsky system. Setting:

$$u(x,t) = \sum_{k=-\infty}^{+\infty} a_k(t) \exp^{(ikx)}, \tag{2}$$

we perform a Galerkin projection to obtain

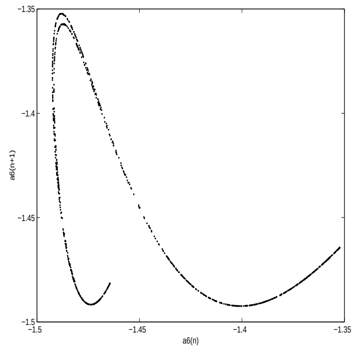
$$\dot{a_k} = (k^2 - \upsilon k^4) + ik \sum_{k=-\infty}^{+\infty} a_k a_{k-m}, \tag{3}$$

Comparing with previous studies,[7] our particular interest is in symmetry unrestricted space.

III Numerical simulation

Guided by the results of the ref.[7] We have carried out a large number of numerical experiments. The size of the truncation ranged from 4 to 16. The dynamics of the system is more complicated as the dissipation parameter υ ($\upsilon = L^2/(2\pi)^2$,L is the length of the system before normalization)decreases. We found some interesting windows are:(i) quasiperiod route to chaos,(ii) a period doubling route to chaos.

To test my jacobian, I calculate the Lyapunov exponents for the system:

(a) 

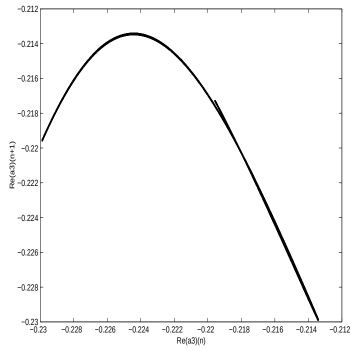
(b) 

Figure 2: Poincare section retuen map. (a)The attractor of the system in antisymmetry case in ref.[7] $\upsilon = 0.02991$, $N = 16$ modes truncation) (b)The attractor of the system in unrestricted symmetry case, 10000 Poincare section returns of a typical trajectory. ($\upsilon = 0.121205$, $N = 8$ modes truncation)

$\upsilon = 0.121205000$,$\lambda_1 = 0.18, \lambda_2 = \lambda_3 = 0.00$,chaotic
$\upsilon = 0.121210000$,$\lambda_1 = 0.16, \lambda_2 = \lambda_3 = 0.00$,chaotic
$\upsilon = 0.121216583$,$\lambda_1 = 0.00, \lambda_2 = \lambda_3 = 0.00$,12-cycle

IV Goal

For the project, we will look at the full space of solutions (the mean velocity is zero) and follow the main idea of the paper.[7] We will use Galerkin projection method to expand the system in a discrete spatial Fourier series. For simulation, we will truncate the ladder of equations to a finite length and also need to fix the parameter values corresponding to the weak turbulence.

Then we shall determine the periodic solutions in the space of Fourier coefficients. We need to develop a numerical program: find initial guesses for periodic points of the full Fourier modes truncation and then determine the cycles by a multi-shooting Newton routine.

Next, we will reconstitute from cycles the unstable spatiotemporally periodic solutions of (1). Having determined the periodic solutions p in the Fourier modes space, we will back to the configuration space and plot the corresponding spatiotemporally periodic solutions.

If enough time remains, we will test the periodic orbit theory by evaluating Lyapunov exponents, escape rates, or another physically motivated global average.

Figure 3: Power spectrum ($\upsilon = 0.121205$, $N = 8$ modes truncation)

References

[1] B.Nicolaenko, B.Scheurer, and R.Temam. Some global dynamical propertiesof the Kuramoto-Sivashinsky equations: nonlinear stability and attractors. Physica D,16,155,1985

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[8] P.Holmes,J.L.Lumley, and G.Berkooz.Turbulence,Cohenrent structure,Dynamical Systems and Symmetry. Cambridge University Press,1996.

Schedule

Oct 23, 2003
Find a suitable truncation length N and a value of υ, which corresponds to the chaotic state.
Oct 30, 2003
Directly find the Lyapunov exponent of the system with the value of υ. Determine the poincare section the system and finish the density plot.
Nov 06, 2003
Study the symbolic dynamics of the system. Finish the program of finding the periodic orbits with the single shooting method. Try to find the periodic orbits with the lowest periods.
Nov 13, 2003
Find all the periodic orbits up to lengh 4.
Nov 20, 2003
If possible, revise the method to find longer periodic orbits (Multipoint shooting method, Variational method)
Nov 27,2003
Calculate the lyapunov exponent of the system with the periodic orbits. Find the energy and the momentum of the system.
Dec 04,2003
Finish the report of the project.
Dec 11,2003
If ok, turn in the the project.