Chapter 3
Maps

Poincaré sections

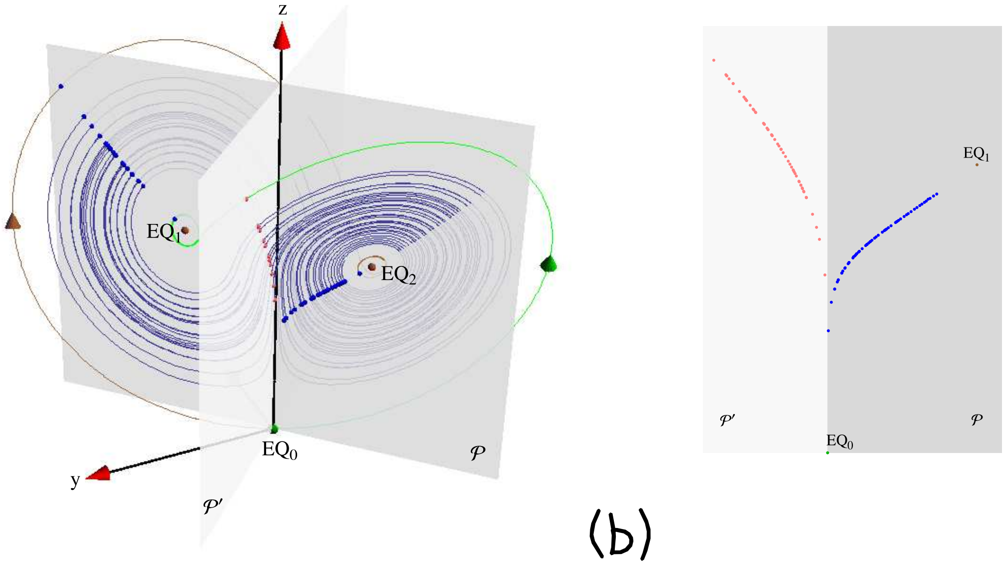
Successive trajectory intersections with a Poincaré section, a d-dimensional set of hypersurfaces $\mathcal{P}$ embedded in the $(d+1)$–dimensional phase space $\mathcal{M}$,

define the Poincaré return map

$$x' = P(x) = f^{\tau(x)}(x), \qquad x', x \in \mathcal{P} .$$

first return function $\tau(x)$ – time of flight to the next section

Sections of Lorenz flow

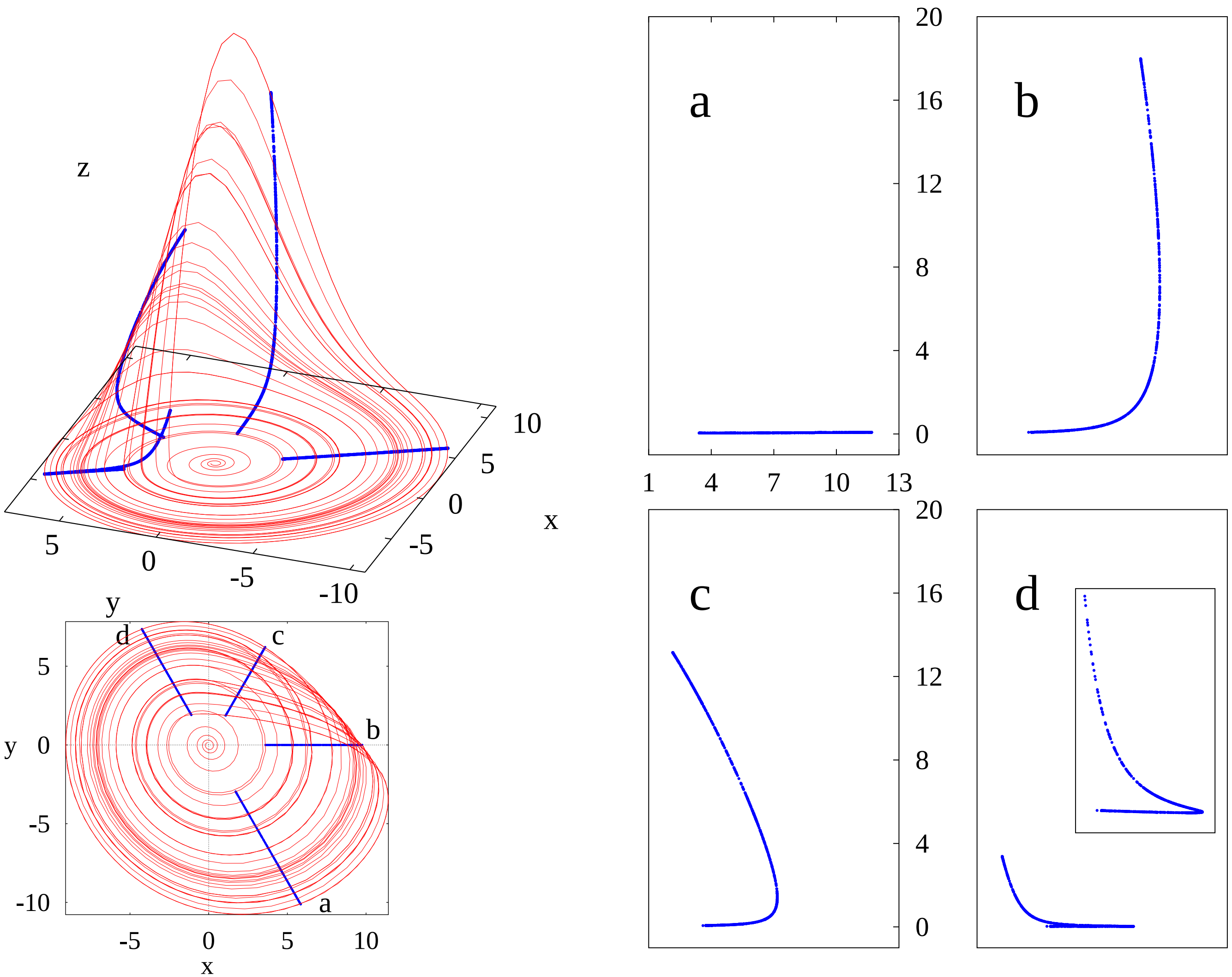


(a) (b)

(a) $y = x$ Poincaré section plane $\mathcal{P}$ through the z axis and both $EQ_{1,2}$ equilibria. The second section $\mathcal{P}'$, through $y = -x$ and the z axis includes the EQ_0 equilibrium,

(b) Poincaré sections $\mathcal{P}$ and $\mathcal{P}'$ laid side-by-side. Note the singularity close to EQ_0.

(E. Siminos)



Poincaré sections, Rössler strange attractor: planes at angles (a) -60^o (b) 0^o, (c) 60^o, (d) 120^o.

Rössler stretch and mix

A line segment [A, B] starts close to the x-y plane, stretching (a) → (b)

flow is expanding

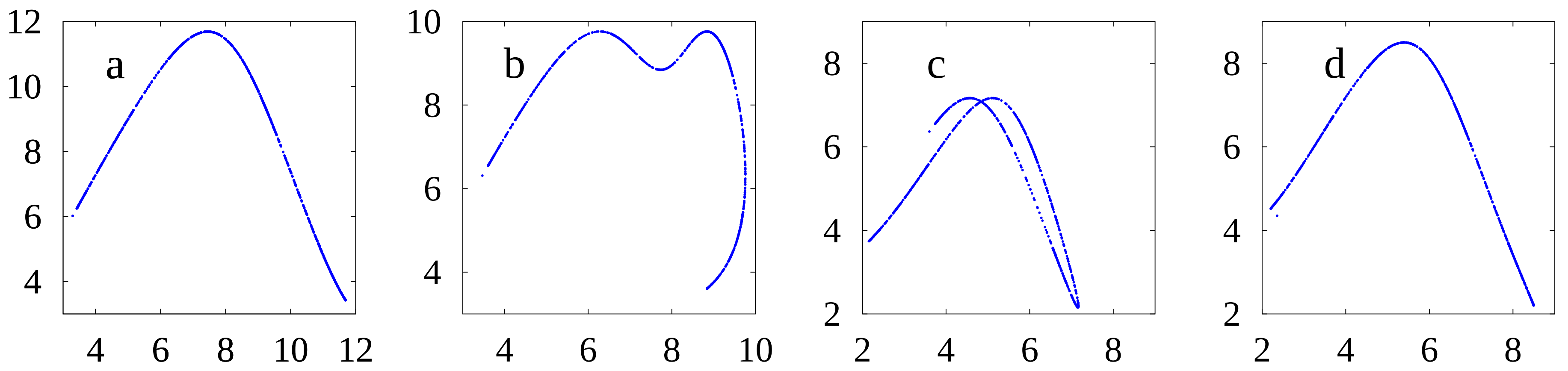
followed by the folding (c) → (d):
the folded segment returns close to the x-y plane

C from the interior mapped into the outer edge
edge point B lands in the interior

flow is mixing

In one Poincaré return the [A, B] interval is stretched, folded and mapped onto itself

Return maps: Poincaré sections projected onto radial distance $R_n \to R_{n+1}$



(a) and (d) nice 1-to-1 return maps

(b) and (c) appear multimodal and non-invertible artifacts of projections $(R_n, z_n) \to (R_{n+1}, z_{n+1})$ onto a 1-dimensional subspace $R_n \to R_{n+1}$