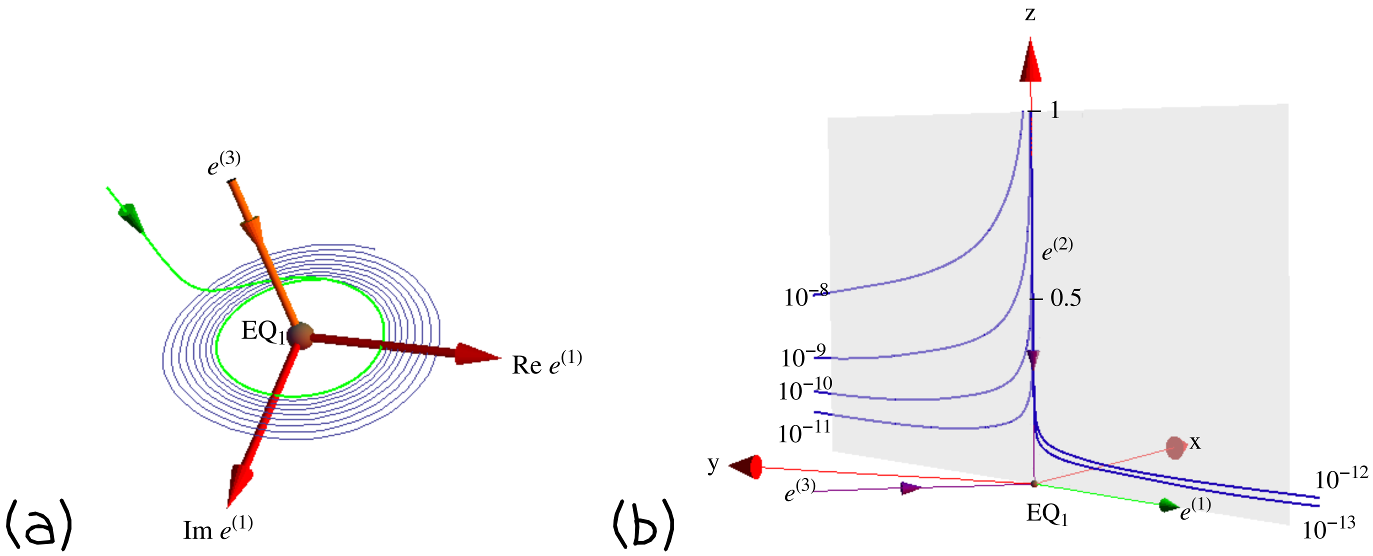
Chapter 11
Qualitative dynamics, for pedestrians

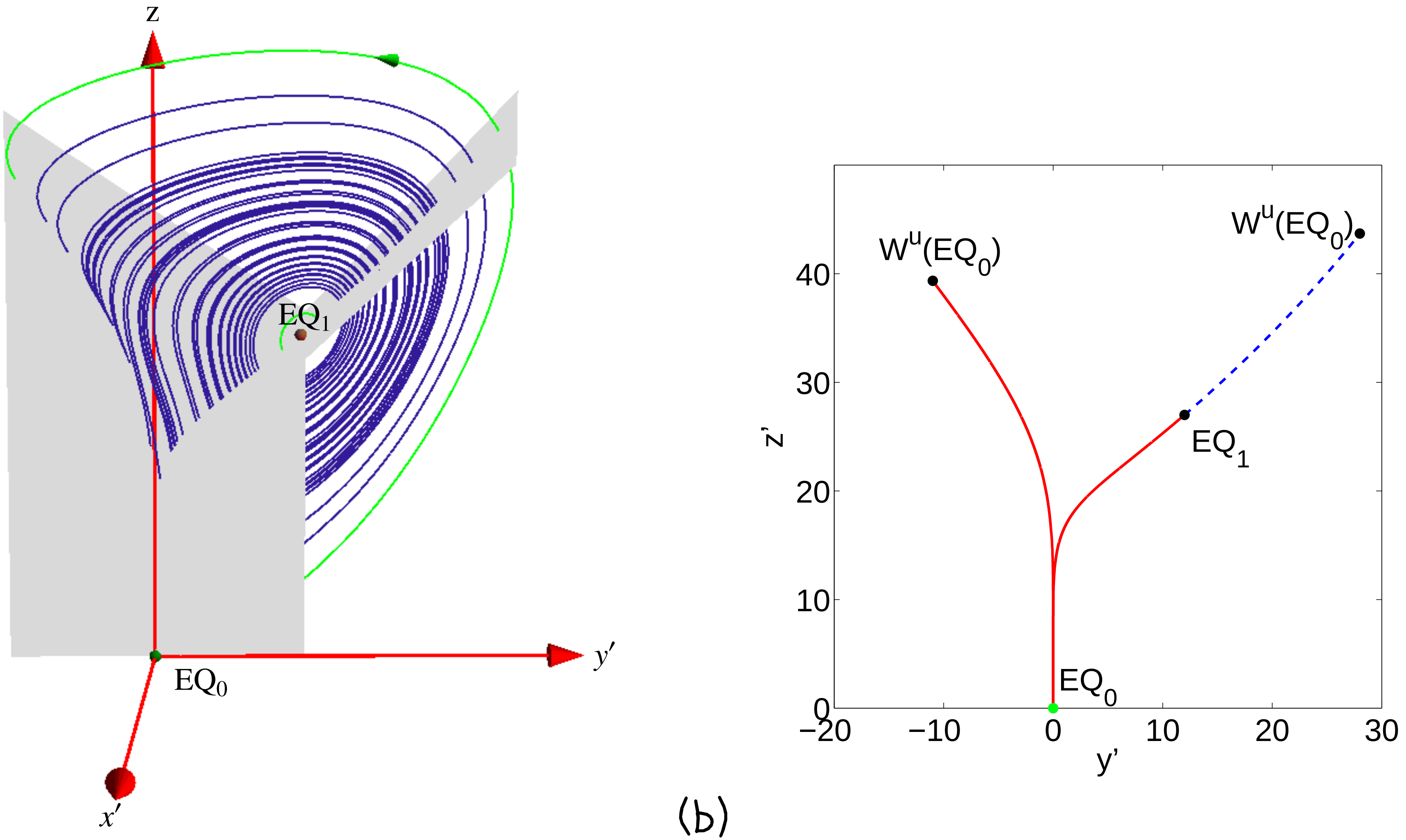
Stability of Lorenz flow equilibria



Lorenz flow: a 1-d return map

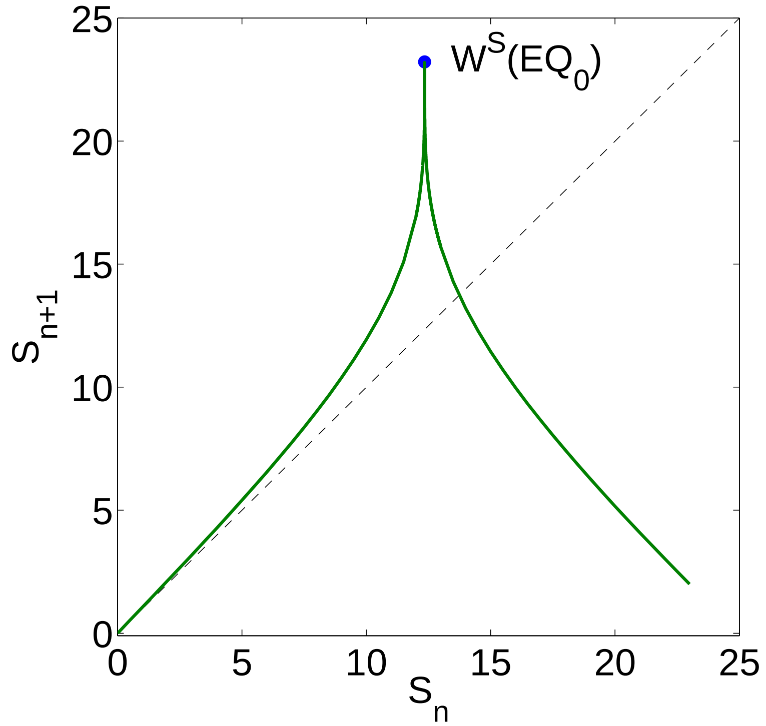
We now deploy the symmetry of Lorenz flow to streamline and complete analysis of the Lorenz strange attractor.

The dihedral $D_1 = \{e, R\}$ symmetry identifies the two equilibria EQ_1 and EQ_2, and the traditional "two-eared" Lorenz flow is replaced by the "single-eared" Van Gogh flow. Furthermore, symmetry identifies two sides of any plane through the z axis, replacing a full-space Poincaré section plane by a half-plane, and the two directions of a full-space eigenvector of EQ_0 by a one-sided eigenvector, see figure ?? (a).



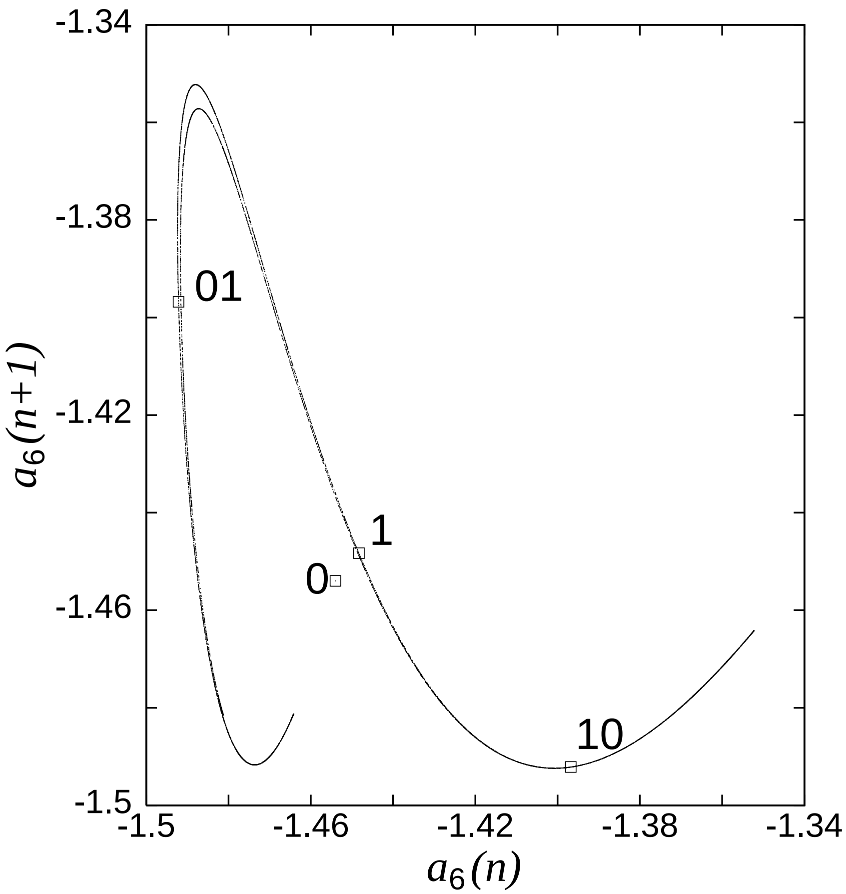
(b)

(a) A Poincaré section of the Lorenz flow in the doubled-polar angle representation, figure ??, given by the $[y', z]$ plane that contains the z-axis and the equilibrium EQ_1. x' axis points toward the viewer. (b) The Poincaré section of the Lorenz flow by the section plane (a); compare with figure ??. Crossings into the section are marked red (solid) and crossings out of the section are marked blue (dotted). Outermost points of both in- and out-sections are given by the EQ_0 unstable manifold $W^u(EQ_0)$ intersections.



The Poincaré return map $s_{n+1} = P(s_n)$ parameterized by Euclidean arclength s measured along the EQ_1 unstable manifold, from x_{EQ1} to $W^u(EQ_0)$ section point, uppermost right point of the blue segment in figure ?? (b). The critical point (the "crease") of the map is given by the section of the heteroclinic orbit $W^s(EQ_0)$ that descends all the way to EQ_0, in infinite time and with infinite slope.

Kuramoto-Sivashinsky



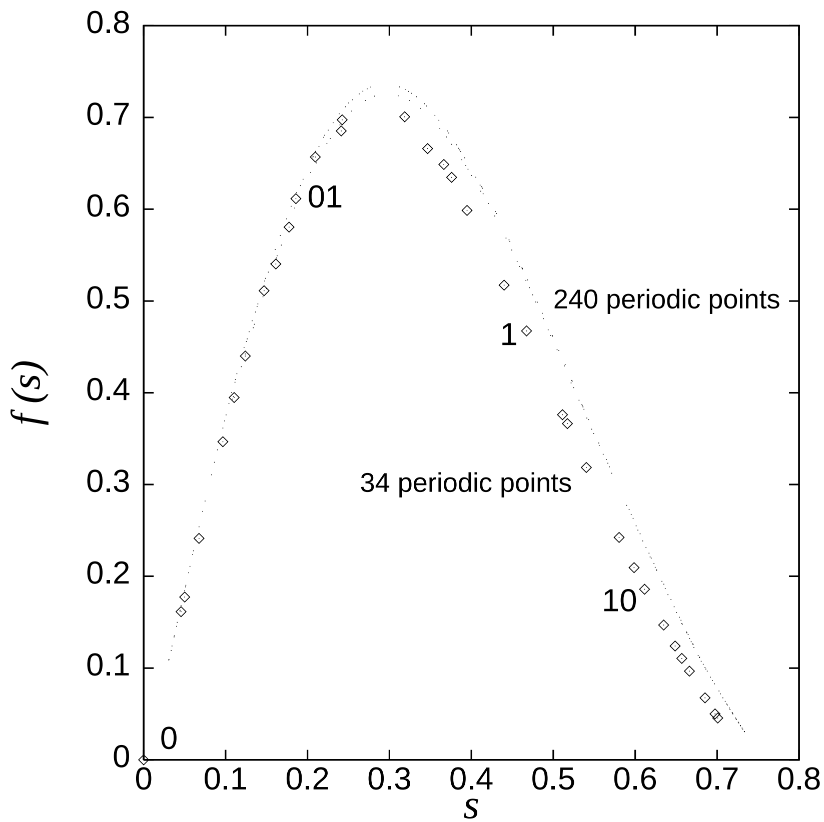
Kuramoto-Sivashinsky

$\tilde{L} \approx 2.889$

very thin Poincaré section

15-d → 15-d Poincaré return map projection on the $[a_6 \rightarrow a_6]$ (or any other) Fourier component is not even $1 \rightarrow 1$.

Intrinsic coordinatization!

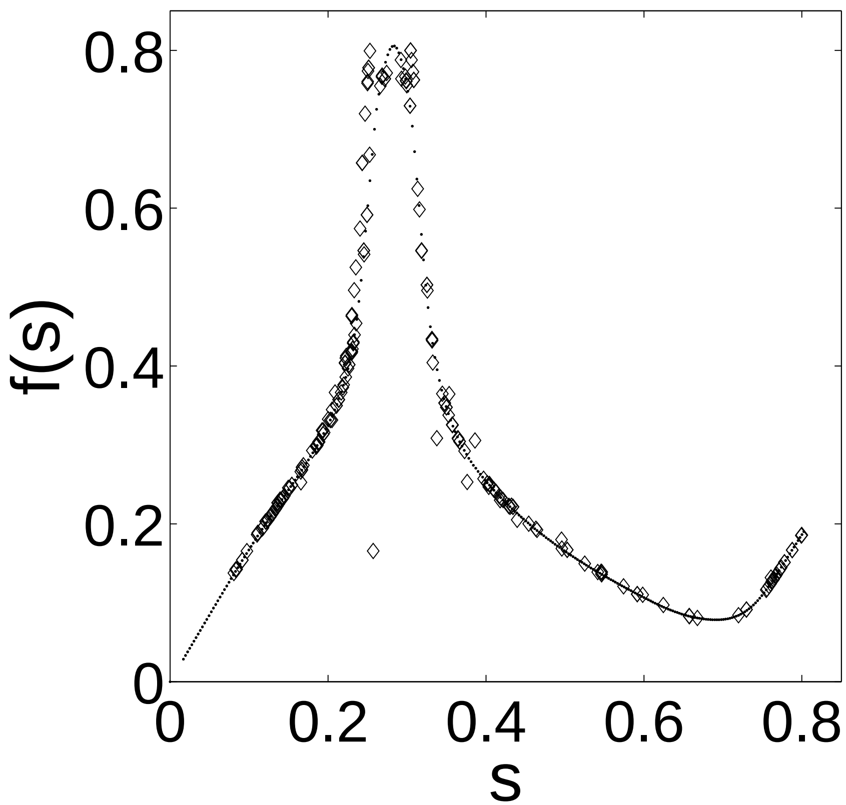


flow visualized as
1-d Poincaré return map
$s \to f(s)$
close returns to unstable manifold of the shortest periodic point

For $\tilde{L} \approx 2.889$ → all periodic solutions up to period n; $\approx 10^3$ unstable recurrent patterns were determined.

Local unstable manifold return maps

Now each thin repelling Smale horseshoe has its local return map $s \to f(s)$ onto the local unstable manifold (shortest cycles indicated):



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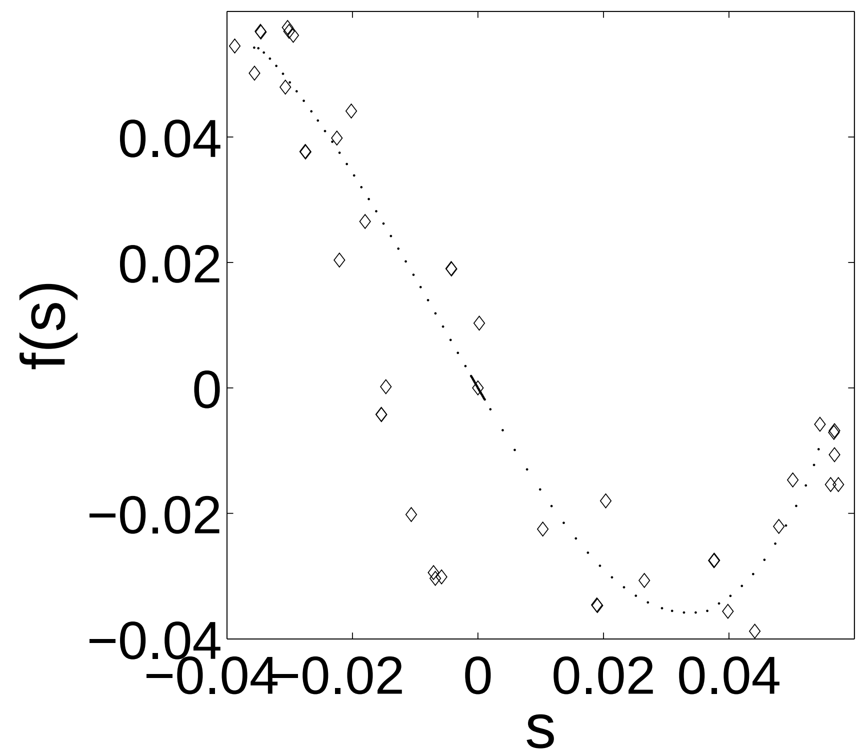

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