

# Chiral orbits

Antônio Endler<sup>1,2,\*</sup> and Jason A.C. Gallas<sup>1,2,†</sup>

<sup>1</sup>*Institut für Computerphysik, Universität Stuttgart, D-70569 Stuttgart, Germany*

<sup>2</sup>*Instituto de Física, Universidade Federal do Rio Grande do Sul, 91501-970 Porto Alegre, Brazil*

(Dated: Draft of January 12, 2005)

We define and analyse “chiral orbits”.

## I. INTRODUCTION

## II. CHIRAL ORBITS

Chiral orbits appear when there is a factor with multiplicity higher than 1 in  $S_k(\sigma)$ . Following Ref. [1], we denote such factors by  $A_k(\sigma)$ ,  $k$  being the periodicity of the orbit. In the Hamiltonian limit and for  $a = 6$  we have

$$S_6(\sigma) = A_6^2(\sigma) B_6(\sigma) C_6(\sigma), \quad (1)$$

$$S_7(\sigma) = A_7^2(\sigma) B_7(\sigma), \quad (2)$$

$$S_8(\sigma) = A_8^2(\sigma) B_8(\sigma) C_8(\sigma), \quad (3)$$

where

$$A_6(\sigma) = \sigma - 2, \quad (4)$$

$$A_7(\sigma) = \sigma^2 - 2\sigma - 6, \quad (5)$$

$$A_8(\sigma) = \sigma^6 - 4\sigma^5 - 52\sigma^4 + 208\sigma^3 + 672\sigma^2 - 2688\sigma - 16, \quad (6)$$

and all other coefficients, not needed here, are given in Ref. [1].

Therefore, chiral orbits exist for the factors ...

### A. Period 6

The orbital equation for the simplest case of Eq. (4),  $\sigma = 2$ , is

$$O_1(x) = x^6 - 2x^5 - 14x^4 + 22x^3 + 62x^2 - 60x - 83, \quad (7)$$

which decomposes into  $O_1(x) = f_1(x) f_2(x)$ , where

$$f_1(x) = x^3 - (1 + \sqrt{3})x^2 - 6x + 5 + 6\sqrt{3}, \quad (8)$$

$$f_2(x) = x^3 - (1 - \sqrt{3})x^2 - 6x + 5 - 6\sqrt{3}. \quad (9)$$

The zeros of  $f_1$  are  $x_1 = -2.409$ ,  $x_2 = 2.101$ ,  $x_3 = 3.040$ , while those of  $f_2$  are  $x_4 = -2.317$ ,  $x_5 = -0.926$ ,  $x_6 = 2.511$ . These orbital points produce two rather different orbits, namely

$$\begin{pmatrix} x_1 & x_6 & x_2 & x_5 & x_3 & x_4 \\ x_4 & x_1 & x_6 & x_2 & x_5 & x_3 \end{pmatrix}. \quad (10)$$

and

$$\begin{pmatrix} x_1 & x_4 & x_3 & x_5 & x_2 & x_6 \\ x_6 & x_1 & x_4 & x_3 & x_5 & x_2 \end{pmatrix}. \quad (11)$$

### B. Period 7

For period-7 there are 18 possible values for the sum of orbital points. In this case  $S_7(\sigma) = A_7(\sigma)^2 B_7(\sigma)$  where

$$A_7(\sigma) = \sigma^2 - 2\sigma - 6, \quad (12)$$

$$B_7(\sigma) = \sigma^{14} + 2\sigma^{13} - 406\sigma^{12} + 288\sigma^{11} + 58540\sigma^{10} - 136808\sigma^9 - 3708984\sigma^8 + 11996864\sigma^7 + 107208320\sigma^6 - 411276032\sigma^5 - 1181332992\sigma^4 + 5368452864\sigma^3 + 901791744\sigma^2 - 11341783040\sigma - 3936915584. \quad (13)$$

For  $A_7(\sigma)$ , the orbital equation due to  $\sigma = 1 + \sqrt{7}$  is

$$P_7(x) = x^7 - (1 + \sqrt{7})x^6 - 2(8 - \sqrt{7})x^5 + (6 + 14\sqrt{7})x^4 + (87 - 22\sqrt{7})x^3 + (21 - 61\sqrt{7})x^2 - 20(8 - 3\sqrt{7})x - 125 + 76\sqrt{7}, \quad (14)$$

while its conjugate, for  $\sigma = 1 - \sqrt{7}$ , is easily obtained from  $P_7(x)$  by replacing  $\pm\sqrt{7} \rightarrow \mp\sqrt{7}$ .

Equação orbital para  $\sigma = 1 + \sqrt{7}$

$$P_1(x) = x^7 - (1 + \sqrt{7})x^6 - 2(8 - \sqrt{7})x^5 + 2(3 + 7\sqrt{7})x^4 + (87 - 22\sqrt{7})x^3 + (21 - 61\sqrt{7})x^2 - 20(8 - 3\sqrt{7})x - 125 + 76\sqrt{7} \quad (15)$$

Soluções:

$$x_1 = -2.363529, x_2 = -2.313389, x_3 = -0.7573026, x_4 = 0.9263635, x_5 = 2.414733, x_6 = 2.727116, x_7 = 3.011758$$

\*E-mail: aendler@if.ufrgs.br

†URL: <http://www.ical.uni-stuttgart.de/~jgallas>

$$\begin{pmatrix} x_1 & x_6 & x_4 & x_5 & x_3 & x_7 & x_2 \\ x_2 & x_1 & x_6 & x_4 & x_5 & x_3 & x_7 \end{pmatrix}. \quad (16)$$

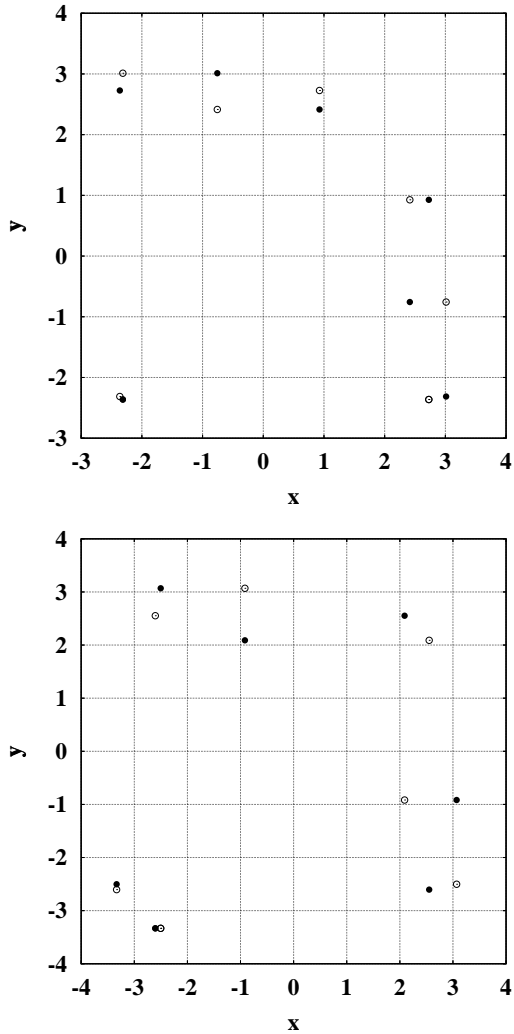


FIG. 1: The two pairs of period-7 chiral orbits.

$$\begin{pmatrix} x_1 & x_2 & x_7 & x_3 & x_5 & x_4 & x_6 \\ x_6 & x_1 & x_2 & x_7 & x_3 & x_5 & x_4 \end{pmatrix}. \quad (17)$$

Equação orbital para  $\sigma = 1 - \sqrt{7}$

$$P_2(x) = x^7 - (1 - \sqrt{7})x^6 - 2(8 + \sqrt{7})x^5$$

$$\begin{aligned} &+ 2(3 - 7\sqrt{7})x^4 + (22\sqrt{7} + 87)x^3 \\ &+ (21 + 61\sqrt{7})x^2 - 20(8 + 3\sqrt{7})x \\ &- 125 - 76\sqrt{7} \end{aligned} \quad (18)$$

Soluções:  $x_1 = -3.332654$   $x_2 = -2.603895$   $x_3 = -2.502687$   $x_4 = -0.9173544$   $x_5 = 2.089252$   $x_6 = 2.552380$   $x_7 = 3.069208$

$$\begin{pmatrix} x_1 & x_3 & x_7 & x_4 & x_5 & x_6 & x_2 \\ x_2 & x_1 & x_3 & x_7 & x_4 & x_5 & x_6 \end{pmatrix}. \quad (19)$$

$$\begin{pmatrix} x_1 & x_2 & x_6 & x_5 & x_4 & x_7 & x_3 \\ x_3 & x_1 & x_2 & x_6 & x_5 & x_4 & x_7 \end{pmatrix}. \quad (20)$$

### C. Period 8

For period-8 there are 30 possible values for the sum of orbital points. Now we find  $S_8(\sigma) = A_8^2(\sigma) B_8(\sigma) C_8(\sigma)$  where

$$A_8(\sigma) = \sigma^6 - 4\sigma^5 - 52\sigma^4 + 208\sigma^3 + 672\sigma^2 - 2688\sigma - 16, \quad (21)$$

$$B_8(\sigma) = \sigma^6 + 8\sigma^5 - 208\sigma^4 - 896\sigma^3 + 12288\sigma^2 + 12288\sigma - 148480, \quad (22)$$

$$C_8(\sigma) = \sigma^{12} - 480\sigma^{10} + 1792\sigma^9 + 64608\sigma^8 - 343296\sigma^7 - 2703200\sigma^6 + 16598016\sigma^5 + 4347648\sigma^4 - 18550272\sigma^3 - 8025600\sigma^2 + 995328\sigma + 553216. \quad (23)$$

Roots of  $A_8(\sigma)$ :

$$-5.299, -4.889, -0.00594, 4.0444, 4.811, 5.338.$$

JACG thanks financial support from the Sonderforschungsbereich 404 of the DFG, Germany. Both authors thank Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil, for support.

[1] A. Endler and J.A.C. Gallas, *Rational reductions in sums of orbital coordinates for a Hamiltonian repeller*, preprint,