Turbulent Transport, Dissipation & Drag

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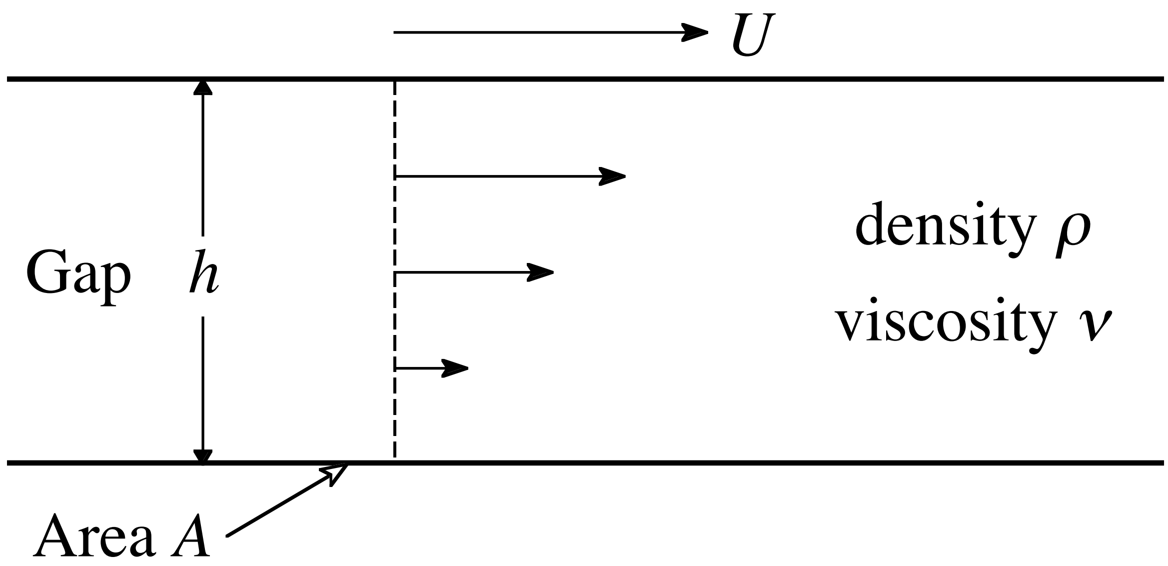
Outline:

1. Phenomenology, physics & philosophy
2. Mathematical models & methods
3. Conclusions & conundrums

Viscosities of familiar fluids:

Fluid	Density (ρ)	Kinematic Viscosity (ν)
Water (0° C)	$1.0 \times 10^3\ kg/m^3$	$1.8 \times 10^{-6}\ m^2/s$
Water (20° C)	$1.0 \times 10^3\ kg/m^3$	$1.0 \times 10^{-6}\ m^2/s$
Air (0° C)	$1.3\ kg/m^3$	$1.3 \times 10^{-5}\ m^2/s$
Air (20° C)	$1.2\ kg/m^3$	$1.5 \times 10^{-5}\ m^2/s$
Glycerine (20° C)	$1.3 \times 10^3\ kg/m^3$	$1.1 \times 10^{-3}\ m^2/s$

Viscosity is the *friction* and the *dissipation* coefficient



Force required to maintain *laminar* flow: $F = \rho \cdot \nu \cdot (U/h) \cdot A$

Power required to maintain *laminar* flow: $P = F \times U = \rho A h \cdot \nu \cdot U^2/h^2$

General relationship: $P_{laminar} \sim$ mass × viscosity × (stirring rate)^2

Example: what is the maximum speed V of your car?

Suppose engine power P = 100 horsepower ≈ 75,000 W.

- If drag due to air is *laminar* friction, then $P = P_{laminar}$.
- Use *spherical approximation* for the car so $P_{laminar} = P_{Stokes} = 6\,\pi\, r\, \rho_{air}\, \nu_{air}\, V^2$
- Therefore, $V_{max} \approx (P/[6\pi\rho_{air}\nu_{air} r])^{1/2}$
- Use $r = 1\ m$ as radius of the sphere.
- $V_{max} = 14{,}000\ m/s = 30{,}000\ mph$!
- *Note:* speed of sound = 350 m/s = 750 mph

Example: What is the maximum speed V of your car?

Suppose engine power P = 100 horsepower ≈ 75,000 W.

- If drag due to air is turbulent dissipation, then $P = P_{turbulent} = c_D \rho_{air} A V^3$.
- c_D is the drag coefficient, depends only only on the *shape* of the car.
- $V_{max} \approx (P / c_D \rho_{air} A)^{1/3}$
- Use $A = 1\ m^2$ and $c_D = .2$ for guesstimate:
- $V_{max} = 66\ m/s \approx$ 140 *mph*
- Compare laminar estimate $V_{max} \approx$ Mach 40!