Spatial simulations of oblique transition in a boundary layer

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(Received 4 January 1994; accepted 1 March 1994)

Simulations of oblique transition in the spatial domain are presented, covering the complete transition process into the turbulent regime. It is conjectured that the three stages identified here and elsewhere are universal for oblique transition in all shear flows: first a nonlinear generation of a streamwise vortex by the oblique waves, second a transient growth of streaks from the vortex by the lift-up effect, and third a breakdown of the streaks due to secondary instability.

We will present an investigation of *bypass* transition, i.e., transition emanating from linear growth mechanisms other than exponential instabilities. This definition is in line with the original idea of Morkovin,[1] but is formulated in view of results of nonmodal transient growth.[2–6] In these investigations it was shown that significant growth of the disturbance energy is possible for certain two- (2-D) and three-dimensional (3-D) disturbances in shear flows at subcritical Reynolds numbers, where the largest growth was obtained for the 3-D perturbations. Physically, the growth is due to the Orr and *lift-up* mechanisms.[7,8] Mathematically, it can be explained by the fact that the linearized Navier–Stokes operator has nonorthogonal eigenfunctions, a necessary condition for subcritical transition to occur.[9]

In the investigation of Henningson *et al.*[10] the lift-up mechanism was found to play an important role in the growth of both infinitesimal and finite amplitude localized disturbances. In the study by Schmid and Henningson[11] temporal simulations starting from a pair of oblique finite amplitude waves were performed. It was found that nonlinearity rapidly excited components with zero streamwise wave number, i.e., streamwise vortices. By the lift-up effect the vortices generated large amplitude low- and high-speed streaks. The breakdown to turbulence, which has recently been found to result from a secondary instability of the streaks,[12] occurs more rapidly than traditional transition initiated by the growth of 2-D waves.

Oblique transition has also been studied in boundary layers on a flat plate.[13,14] It was found that the "streamwise vortex mode" played an important role and that the initial amplitude necessary to trigger transition was lower than for comparable secondary instability scenarios. This scenario has also been found in a study of a compressible confined shear layer,[15] where the oblique waves appeared naturally from noise introduced at the inflow boundary.

In the present study the oblique transition scenario has been simulated spatially for a zero-pressure gradient incompressible boundary layer. We will use a numerical simulation program solving the full 3-D incompressible Navier–Stokes equations.[16] The program uses Fourier–Chebyshev spectral methods, and has recently been modified to handle spatial development of disturbances in boundary-layer flows. In a fringe region a forcing term was added to the Navier–Stokes equations. It was implemented such that the disturbances flowing out of the box were eliminated and the flow returned to its laminar state. In the fringe region wave disturbances can also be generated, simulating a vibrating ribbon. This technique is similar to that of Bertolotti *et al.*[17]

The inflow conditions for the present simulation consists of the Blasius mean flow plus a pair of oblique waves, each with an amplitude A (based on the maximum RMS of the streamwise velocity) of 0.01. They are taken as the least-damped Orr–Sommerfeld mode for $\omega_0=0.08$ ($F_0=\omega_0/R=200\times10^{-6}$) and $\beta_0=0.192$, excluding the associated normal vorticity. Here ω_0 and $\pm\beta_0$ are the angular frequency and spanwise wave numbers of the generated waves. The Reynolds number at the inflow ($R=U_\infty\delta_0^*/\nu$) is 400, based on the inflow displacement thickness (δ_0^*) and free-stream velocity (U_∞), which in the following are used to nondimensionalize all quantities. The inflow position will in the following be denoted x_0.

Two calculations of the same scenario were performed. The first used $480\times97\times80$ modes in the streamwise, normal, and spanwise directions, respectively, and the second used $720\times121\times120$ modes. (Note that spanwise symmetry was assumed and that dealiasing, using the 3/2 rule, was also applied in the horizontal directions.) As a test of the convergence, four maxima in the streamwise shear in the outer part of the boundary layer were compared [$x-x_0\approx200,250,300,350$, $y\approx5$ in Fig. 2(b)]. The differences in the

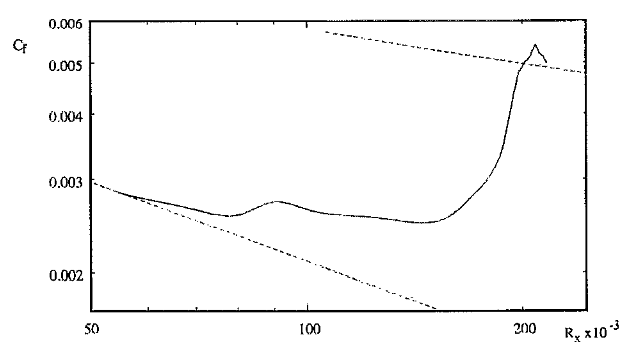

FIG. 1. Coefficient of friction $c_f=2\tau_w/\rho U_\infty^2$ (solid curve), τ_w is the averaged wall shear stress. $R_x=xU_\infty/\nu$ where x is the distance from the leading edge. Lower dashed line shows the value for a laminar Blasius boundary layer ($0.664\ R_x^{-1/2}$) and the upper curve is the turbulent friction $0.370(\log R_x)^{-2.584}$ by Shultz-Grunow.

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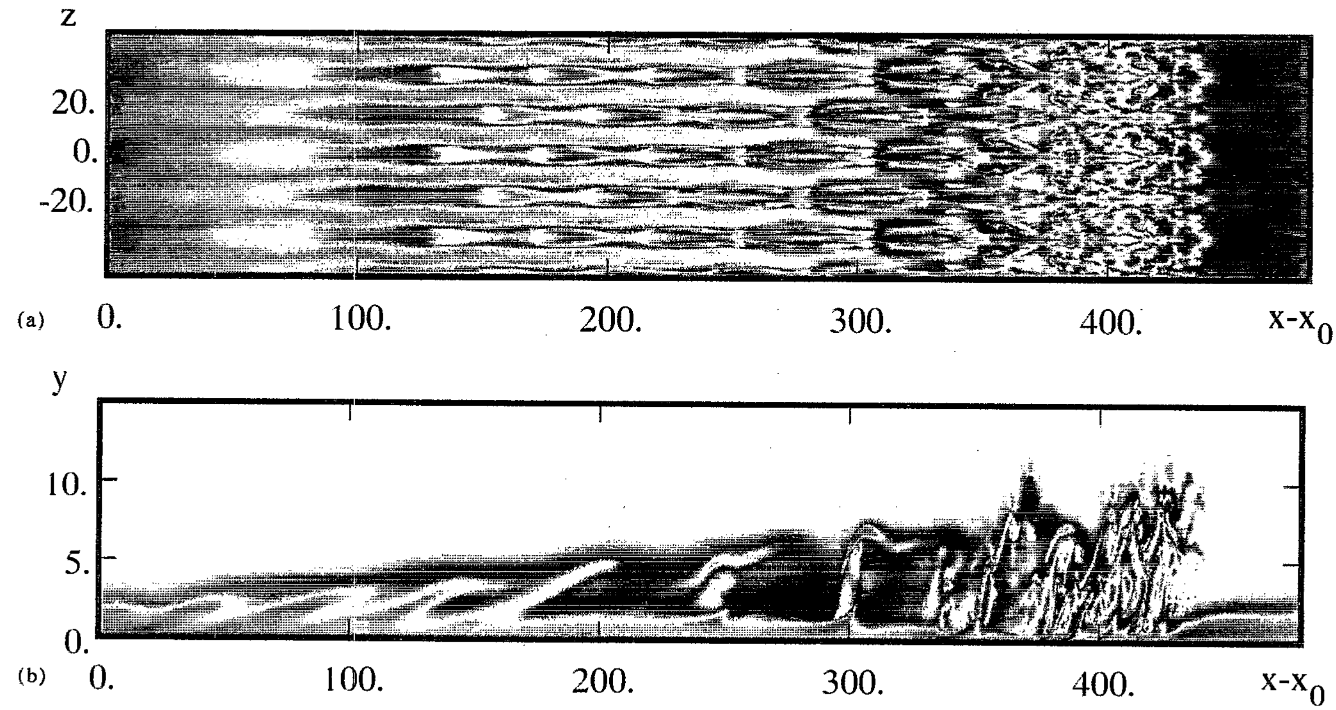

FIG. 2. Instantaneous velocity fields. (a) Streamwise velocity at $y=2.93$. Values range from red at 0.34 to blue at 1.08. (b) Streamwise shear at $z=0$. Values range from blue at -0.18 to red at 1.9. Note the fringe region starting at $x-x_0$=408, at the right part of the computational box.

values were below 1% in the four maxima, although the position of the last maximum has changed slightly. The higher resolution corresponds to a grid step of 14 wall units in the streamwise direction, 6 for the spanwise, and 4 for the largest step in the wall normal direction, based on the wall friction in the turbulent region.

Figure 1 shows the development of the coefficient of friction ($c_f=2\tau_w/\rho U_\infty^2, \tau_w$ is the time- and spanwise-averaged wall shear stress) for the simulation. It is evident that the simulation captures the complete transition process, all the way into the turbulent regime.

Figure 2 shows the breakdown to turbulence to the two oblique waves generated at the inflow boundary. In Fig. 2(a), which shows the streamwise velocity in a wall-parallel plane, the appearance of streamwise streaks is observed at about $x-x_0=50$. The streaks subsequently grow to a large amplitude and become unstable to nonstationary disturbances, resulting in a breakdown to turbulence at about $x-x_0$=350. Figure 2(b) shows the streamwise shear in a side view of the boundary layer. Shear layers are seen to intensify and become unstable prior to the breakdown.

Figure 3 shows the energy in some of the excited Fourier components during the transition process. At the inflow only the $(1,\pm 1)$ components are excited. They show a rapid initial growth similar to that in the simulations by Schmid and Henningson,[11] who also set the initial normal vorticity to zero. The (0,0), $(0,\pm 2)$, (2,0), and $(2,\pm 2)$ components subsequently increase due to nonlinear effects, since they are directly generated by the $(1,\pm 1)$ modes through the quadratic nonlinearity. The $(0,\pm 2)$ components grow more rapidly than the other modes and continues to grow until about $x-x_0=100$. The latter part of this growth was found by Schmid and Henningson[11] to be due to a linear forcing of the streak (u component) from the vortex (v,w components) for the same wave number. A second phase of rapid growth starts for modes with nonzero ω, eventually completing the transition process. This growth can best be described as a secondary instability on the base flow with a spanwise variation given by the $(0,\pm 2)$ streaks. A similar rapid growth of oblique modes from a state of streamwise streaks was found for transition in plane Couette flow by Kreiss *et al.*[12]

In order to put the present simulation in perspective, data from a number of recent spatial simulations have been compiled in Table I. The transition process in the present simulations occupies about the same streamwise domain as in the simulations of secondary-instability-induced breakdown by Kloker and Fasel,[18] in spite of the exponential growth of the

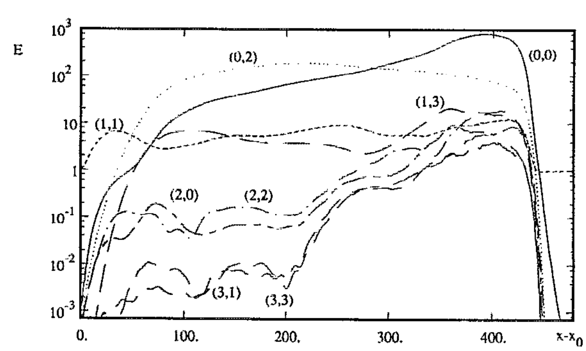

FIG. 3. Energy in Fourier components with frequency and spanwise wave number ($\omega/\omega_0,\beta/\beta_0$) as shown. The curves are normalized such that the energy of the (1,1) mode at inflow is set to unity.

TABLE I. Comparison of recent spatial simulations of instability and transition. The last column indicates whether the simulation included the complete transition region; x_0 is the position of the disturbance generator and x_E is the end of the simulated region.

Investigator	R	R_{x_0}	R_{x_E}	$A_{2\text{-D}}$	$A_{3\text{-D}}$	ω_0	β_0	Transition
Present	400	54 000	220 000	-	0.01	0.080	0.192	yes
JSC[15]	733	182 000	447 000	0.0048	0.000 014 5	0.091	0.242	no
JSC[15]	900	238 000	489 000	-	0.01	0.0774	0.2	no
KF[18]	679	155 000	304 000	0.03	0.002	0.075	0.29	yes
SY[19]	1260	532 000	1 390 000	0.01	noise	0.095	-	no

2-D mode and higher input amplitude in the latter case. This is accentuated by the results of Spalart and Yang[19] who simulated an even larger domain by following a streamwise periodic box, accounting for the growth of the boundary layer in an approximate manner. In spite of covering a larger Reynolds number range their simulations did not reach the turbulent state.

In the present investigation the wave amplitude at the inflow is low, resulting in a long growth region before breakdown. This initial amplitude represents the lowest amplitude disturbance of the chosen form, giving transition in this computational box. In a simulation with $A=0.0086$ the secondary instability was not strong enough, and thus no transition occurred. For the same initial amplitude Joslin *et al.*[15] did not find that the growth was sufficiently rapid to cause transition within their computational box, although the domain was longer and the inflow at a higher Reynolds number than the present study. The reason may be their use of complete eigenmodes as inflow condition (i.e., including the normal vorticity part of the eigenmode), which implies that they do not have the rapid transient growth of the oblique $(1,\pm1)$ modes seen in the present case.

The oblique transition scenario in the boundary layer is quite similar to that seen in channel flow,[11] and the streaks seem to break down due to the same secondary instability mechanism.[12] In light of these findings, and those of other investigations discussed here, we conjecture that the following three stages occurs during oblique transition in shear flows:

(i) Initial nonlinear generation of a streamwise vortex by the two oblique waves.

(ii) Generation of streaks from the interaction of the streamwise vortex with the mean shear by the lift-up effect.

(iii) Breakdown of the flow due to a secondary instability of the streaks, when these exceed a threshold amplitude.

Note that if the amplitude of the inflow disturbance is large enough the breakdown may be so rapid that the second and the third stage overlap.

ACKNOWLEDGMENTS

The computer time for the present simulations were provided by the National Center for Supercomputing Applications, Urbana, Illinois and the Pittsburgh Supercomputer Center, Pittsburgh, Pennsylvania. The research was funded by The Swedish Research Council for the Engineering Sciences (TFR) and the Swedish National Board for Industrial and Technical Development (NVTEK).

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