

The problem I have is about the derivation of the fact that the escape rate is the leading eigenvalue of the Perron-Frobenius-Operator. In order to calculate the escape rate, one has to examine the asymptotic behaviour of the quantity

$$\Gamma_n = \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} dx \int_{\mathcal{M}} dy \delta(y - f^n(x)). \quad (1)$$

Of course the  $dx$ -integral is nothing but the Perron-Frobenius-Operator  $\mathcal{L}^n$  acting on an uniform initial density  $i(x) = 1 \forall x \in \mathcal{M}$ :

$$\Gamma_n = \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} dy (\mathcal{L}^n i)(y). \quad (2)$$

If I understood it correctly, you argue in the following way: the initial density  $i(x)$  can be expanded in terms of eigenfunctions of  $\mathcal{L}$ ,

$$i(x) = \sum_{\alpha} c_{\alpha} \varphi_{\alpha}(x), \quad (3)$$

and therefore, for large  $n$ ,  $\Gamma_n$  is dominated by  $\lambda_0$ , the leading eigenvalue of  $\mathcal{L}$ :  $\Gamma_n \sim \lambda_0^n$  as  $n \rightarrow \infty$ .

My first and most important question is the following: is the decomposition (3) really possible in an open system?

If trajectories can escape and the invariant set  $\Lambda$  is only a subset of  $\mathcal{M}$  of zero Lebesgue measure, I think the eigenfunctions  $\varphi_{\alpha}$  must be zero almost everywhere. Why? The eigenvalue condition

$$\begin{aligned} (\mathcal{L}^n \varphi_{\alpha})(y) &= \int_{\mathcal{M}} dx \delta(y - f^n(x)) \varphi_{\alpha}(x) \\ &= \lambda_{\alpha}^n \varphi_{\alpha}(y) \end{aligned} \quad (4)$$

yields that  $\varphi_{\alpha}$  can have nonzero values only on the set  $\cap_{k=0}^n f^k(\mathcal{M})$ . This set becomes arbitrary small for large  $n$ , and (4) holds for every  $n$ , if  $f$  is invertible, it holds even for negative  $n$ . Then, all the  $\varphi_{\alpha}$  must be concentrated on the invariant set  $\Lambda$ , or at least on the set  $\Lambda_{+}^{\infty} := \cap_{k=0}^{\infty} f^k(\mathcal{M})$ , and it is impossible to expand  $i(x) = 1 \forall x \in \mathcal{M}$  in terms of the eigenfunctions  $\varphi_{\alpha}$ .

So how does it work? Do I have to think of the  $\varphi_{\alpha}$  as functions that are a little bit smoothed around  $\Lambda_{+}^{\infty}$ ? For large  $n$ , only points close to  $\Lambda_{+}^{\infty}$  contribute to the  $dy$ -integral in (1). Or am I dead wrong?

If this problem is solved, there are two questions remaining. Is  $\{\varphi_{\alpha}\}$  a basis for a (properly chosen) function space? And can I be sure that the coefficient  $c_0$  in (3) isn't zero? Otherwise  $\lambda_0$  would not be dominating.

Thank you very much for looking at this.