

Program to compute periodic orbits of forced pendulum type flows

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The program `period.c` computes the periodic orbits of the forced pendulum model

$$H(p, x, t) = \frac{p^2}{2} - \varepsilon(\cos x + \cos(x - t)). \quad (1)$$

Procedures Besides trivial procedures (`identite`, `annulmat`, `annulvec`, `mult`, `egal`), here is a list of the procedures of the program

- `ludcmp`, `lubksb` - procedures from Numerical Recipes to invert a matrix
- `rk4` - procedure (from Numerical Recipes) to integrate the equations of motion using fourth-order Runge-Kutta method. The integration step is `h` which is chosen to be $2\pi/N$, $N = 2000$.
- `flow` - equations of motion associated to the variable (x, p, t)

$$\begin{aligned} \dot{x} &= p, \\ \dot{p} &= -\varepsilon(\sin(x) + \sin(x - t)), \\ \dot{t} &= 1. \end{aligned}$$

Here `eps` = 0.0275. In the program, the indices 1, 2, 3 correspond to x , p , t respectively.

- `pmap` - procedure that computes the Poincaré map and the stability matrix
- `main` - this is the main procedure. In order to compute the periodic orbit with period `Q`, we use multiple shooting method (Newton-Raphson method on an extended space). Starting from initial conditions `(x1, x2, x3)` which are the positions of the periodic orbit in the integrable case $\varepsilon = 0$, It first computes the matrix DF (see lecture notes <http://cns.physics.gatech.edu/~chandre/lecture12.pdf>, page 9), called `Ain` in the program. Then by inverting this matrix, we obtain a better approximation to the periodic orbit (if lucky enough...). This procedure is iterated until some precision `delta` is reached, or the procedure is ended because the precision diverges.