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Farey Organization of the Fractional Hall Effect

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Abstract

We conjecture that the fractional quantized Hall effect Landau-level occupation factors are organized by the Haldane tree. This is a variant of the Farey tree, suitably refined to account for the absence of forbidden states.

Haldane [1] has conjectured that the fractional Hall effect occurs for Landau-level occupation factors whose continued fraction representation is of form

$$[m, \pm p, \pm q, \pm r, \ldots] = \cfrac{1}{m \pm \cfrac{1}{p \pm \cfrac{1}{q \pm \cfrac{1}{r \pm \ldots}}}} \tag{1}$$

where m is odd, and $p, q, r, \ldots$ are even.

In this model the most stable "incompressible fluid" states correspond to fractions with smallest denominators. When the magnitude of an effect decreases with increasing denominator, it is often profitable [2] to investigate whether the effect is organized by the corresponding Farey tree.

The Farey mediant [3] interpolates between two rationals, $P/Q \oplus R/S = (P+R)/(Q+S)$, yielding the fraction with the smallest denominator lying between P/Q and R/S. The Farey tree is obtained by starting with the ends of the unit interval written as 0/1 and 1/1, and interpolating by means of Farey mediants. The first level of the Farey tree is 1/2, the second 1/3, 2/3, the third 1/4, 2/5, 3/5, 4/5 and so forth. The following alternative construction of the Farey tree makes the self-similarities more explicit. Replace each Farey number by its continued fraction representation $P/Q = [j, k, l, \ldots, m]$, with $j, k, l, \ldots, m$ positive integers. The next level of the Farey tree is obtained by replacing the "last 1" in the continued fraction by either 2 or 1/2:

$$\begin{array}{ccc} & [j, k, l, \ldots, m] & \\ \swarrow & & \searrow \\ [j, k, l, \ldots, m+1] & & [j, k, l, \ldots, m-1, 2] \end{array} \tag{2}$$

The continued fraction representation shows explicitly that each of the Farey tree is similar to the entire tree, and suggests the form of the scaling laws for associated physical quantities.

The rule (2) requires positive coefficients, so it does not apply to Haldane's continued fractions (1). If negative coefficients are allowed, there is an ambiguity in the notation for continued fractions, as

$$[\ldots, k, 2] = [\ldots, k+1, -2] \tag{3}$$

This ambiguity is removed by requiring that whenever it arises, one takes the alternative which satisfies the Haldane's conditions (1). After some algebra, the rule (2), is replaced by a pair of rules:

(a) if $p \neq -2$, then

$$\begin{array}{ccc} & [\ldots, \pm p] & \\ \swarrow & & \searrow \\ [\ldots, \pm(p+1)] & & [\ldots, \pm(p-1), 2] \text{ or } [\ldots, \pm p, -2] \end{array} \tag{4}$$

(b) if $p = -2$, then

$$\begin{array}{ccc} & [\ldots, -2] & \\ \swarrow & & \searrow \\ [\ldots, -2, -2] & & [\ldots, -3] \end{array} \tag{5}$$

The Haldane tree contains three kinds of fractions: Haldane forbidden, Haldane allowed ending with -2, and Haldane allowed not ending with -2. As in the case of the Farey tree, each branch is similar to the entire tree, but now there are three kinds of branches, corresponding to the three kinds of fractions. The self-similarities of the Haldane tree suggest existence of scaling laws which would relate strengths of the associated physical effects (Hall resistance, sheet resistance, etc.). At present, a reliable method for computing such effects is lacking, but if such method becomes available, the Haldane tree can be used as a guide in searching for the scalings in the fractional Hall effect.

References

1. Haldane, F. D. M., Phys. Rev. Lett. 51, 605 (1983).
2. See, for example, the contribution of S. Ostlund and S. -h. Kim in this issue of Physica Scripta.
3. Hardy, G. H. and Wright, E. M., Theory of Numbers, Oxford University Press, Oxford (1938).