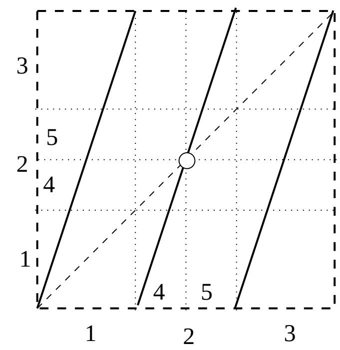

(a) **(b)** **(c)** **(d)**

Figure S.17: (a) (b) A partition of the unit interval into three or five intervals, labeled by the order along the unit interval $\mathcal{A} = \{\mathcal{M}_1,\, \mathcal{M}_2 = \mathcal{M}_4 \cup (\frac{1}{2}) \cup \mathcal{M}_5,\, \mathcal{M}_3\}$. The partition is Markov, as the critical point is also a fixed point. (c) the Markov graph for this Markov partition.

Chapter 25. Deterministic diffusion

Solution 25.1 - Diffusion for odd integer Λ. *Consider first the case $\Lambda = 3$, illustrated in figure S.17. If $\beta = 0$, the dynamics in the elementary cell is simple enough; a partition can be constructed from three intervals, which we label $\{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3\}$, with the alphabet ordered as the intervals are laid out along the unit interval. The Markov graph is figure S.17 (c), and the dynamical zeta function is*

$$1/\zeta|_{\beta=0} = 1 - (t_1 + t_2 + t_3) = 1 - 3z/\Lambda\,,$$

with eigenvalue $z = 1$ as required by the flow conservation.

However, description of global diffusion requires more care. As explained in the definition of the map (25.9), we have to split the partition $\mathcal{M}_2 = \mathcal{M}_4 \cup (\frac{1}{2}) \cup \mathcal{M}_5$, and exclude the fixed point $f(\frac{1}{2}) = \frac{1}{2}$, as the map $\hat{f}(\hat{x})$ is not defined at $\hat{f}(\frac{1}{2})$. (Are we to jump to the right or to the left at that point?) As we have $f(\mathcal{M}_4) = \mathcal{M}_1 \cup \mathcal{M}_4$, and similarly for $f(\mathcal{M}_5)$, the Markov graph figure S.17 (d) is infinite, and so is the dynamical zeta function:

$$1/\zeta = 1 - t_1 - t_{14} - t_{144} - t_{1444} \cdots - t_3 - t_{35} - t_{355} - t_{3555} \cdots .$$

The infinite alphabet $\mathcal{A} = \{1, 14, 144, 1444 \cdots 3, 35, 355, 3555 \cdots\}$ is a consequence of the exclusion of the fixed point(s) x_4, x_5. As is customary in such situations (see sect. 22.3.1, *exercise 18.10, and chapter 22,* inter alia*), we deal with this by dividing out the undesired fixed point from the dynamical zeta function. We can factorize and resum the weights using the piecewise linearity of (25.9)*

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$$1/\zeta = 1 - \frac{t_1}{1-t_4} - \frac{t_3}{1-t_5}\,.$$

The diffusion constant is now most conveniently evaluated by evaluating the partial derivatives of $1/\zeta$ as in (18.19)

$$\begin{aligned}
\langle T \rangle_\zeta &= -z\frac{\partial}{\partial z}\frac{1}{\zeta} = 2\left(\frac{t_1}{1-t_4} + \frac{t_1 t_4}{(1-t_4)^2}\right)\Bigg|_{z=1,\beta=0} = \frac{3}{4} \\
\left\langle \hat{x}^2 \right\rangle_\zeta\Big|_{z=1,\beta=0} &= 2\left(\frac{\hat{n}_1(\hat{n}_1+\hat{n}_4)\Lambda^2}{(1-1/\Lambda)^2} + 2\frac{\hat{n}_4^2/\Lambda^3}{(1-1/\Lambda)^3}\right) = \frac{1}{2}
\end{aligned} \qquad \text{(S.66)}$$

yielding $D = 1/3$, in agreement with in (25.21) for $\Lambda = 3$.

Solution 25.6 - Accelerated diffusion. Suggested steps

1. *Show that the condition assuring that a trajectory indexed by (ϕ, α) hits the (m, n) disk (all other disks being transparent) is written as*

$$\left| \frac{d_{m,n}}{R} \sin(\phi - \alpha - \theta_{m,n}) + \sin\alpha \right| \leq 1 \tag{S.67}$$

   where $d_{m,n} = \sqrt{m^2 + n^2}$ and $\theta_{m,n} = \arctan(n/m)$. You can then use a small R expansion of (S.67).

2. *Now call j_n the portion of the state space leading to a first collision with disk $(n, 1)$ (take into account screening by disks $(1, 0)$ or $(n - 1, 1)$). Denote by $J_n = \bigcup_{k=n+1}^{\infty} j_k$ and show that $J_n \sim 1/n^2$, from which the result for the distribution function follows.*