Chiral orbits

Antônio Endler,[1,*] Jason A.C. Gallas,[1,2,†] and Predrag Cvitanović[3,‡]

[1] *Instituto de Física, Universidade Federal do Rio Grande do Sul, 91501-970 Porto Alegre, Brazil*
[2] *Institut für Computerphysik, Universität Stuttgart, D-70569 Stuttgart, Germany*
[3] *Georgia Inst Technol, Sch Phys, Atlanta, GA 30332 USA*

(Dated: Draft of January 25, 2005)

Chirality is a key concept pervading physics, chemistry and with enormous impact in biology. Traditionally, chirality is defined mathematically in a geometrical way, using symmetry operations centered on reflections about symmetry planes. In field theory it appears as .... Here we show a new class of orbits, chiral orbits, exists in profusion in the skeleton of infinite unstable periodic orbits embedded in chaotic attractors which are or paramount importance in the interpretation of quantum-mechanical spectra of systems whose classical counterpart exhibit chaotic behavior. These orbits are a natural consequence of the dynamics, may be described by simple maps and imply simplifications of summations of trace formulas used to describe atomic spectra semi-classically.

I. INTRODUCTION

One of the major themes in drug design, discovery and development is chirality, the property shown by two structures that are mirror images of each other. The importance of chiral activity is probably best known from the thalidomide tragedy of the 1960s, when administration of a mixture of both left- and right-handed versions (enantiomers) of the drug to pregnant women caused a number of birth defects [1]. In physics, two examples suffice to show the great importance of chirality both in applications and at a more fundamental level.

First, the discovery of superfluidity in ^{3}He was followed by evidence that for other novel types of superconductivity for which the Cooper pairs may also exist in states with nontrivial internal structure. Therefore it is important to find probes sensitive to the internal degrees of freedom of the Cooper pairs in condensates with complicated broken symmetries, such as heavy fermions and high-T_c superconductors. add references here

Second, Kawamura and coworkers [6] have made large-scale numerical investigations of the chiral driven ordering mechanisms which they postulated to explain freezing in Heisenberg spin glasses with no anisotropy. It was clearly shown that chirality is a primary ingredient of spin glass ordering in vector systems. Chirality is here a "hidden" parameter and no technique was known to monitor it experimentally until Tatara and Kawamura proposed that the Extraordinary Hall effect should present a direct signature of the chiral susceptibility [7]. This was recently measured directly [8] .. bla bla bla.... polish this here.

Second, the presence of infinite dimensional chiral symmetries is central to the study of two dimensional conformal field theories. These theories, called *chiral algebras*, are formed by the purely holomorphic and purely antiholomophic fields, respectively. How the field content of a conformal field theory is organized into representations of the chiral algebras is encoded in the fusion ring. As pointed out by Kausch[2], the task of solving and classifying general conformal field theories can be split into two separate problems: to find chiral algebras of conformal field theories and to determine possible fusion rings of chiral algebras. Such program has been most succesful for the Virasoro minimal series [3], where the fields fall into infinitely many representations of the Virasoro algebra. This has subsequently enabled the complete characterization of the field content of such theories[2, 4]. Beilinson and Drinfeld translated the key concept of chiral algebras of 2D conformal field theory into the language of algebraic geometry. This bridge allows one to recognize chiral algebras to be crucial for certain questions of representation theory and arithmetics. In particular, they seem to present a new approach to the geometric version of Langlands correspondence introduced by Drinfeld some 20 years ago [5]. The above paragraph is too long!!... Condense the above physical justification/motivations ...

The purpose of this letter is ......

In addition to the well-known sensitivity to initial conditions leading to chaos in dynamical systems, we report an additional type of sensitivity to initial conditions seen in multidimensional systems: *phase degeneracy*. Phase degeneracy means that orbital points may be combined in more than one way to produce orbital motions. These implies that degeneracies of equations of motion may be far more numerous than believed. Additional consequences of phase sensitivy are discussed.

Strongly anharmonic and translationally invariant systems in arbitrary dimensions, exhibit a class of time periodic and stable solutions carrying an energy flow as well as the standard plane waves which are special cases. In general, the spatial distribution of these energy flows is very inhomogeneous and form arbitrarily complex networks of channels and vortices. These solutions are constructed from arbitrary, finite, or infinite clusters of breathers (multibreathers) with twisted phases. Examples of these solutions are numerically calculated in several one and two-dimensional nonlinear models.

*E-mail: aendler@if.ufrgs.br
†E-mail: jgallas@ica1.uni-stuttgart.de
‡E-mail: predrag.cvitanovic@physics.gatech.edu

II. CHIRAL ORBITS

Chiral orbits appear when there is a factor with multiplicity higher than 1 in $S_k(\sigma)$. Following Ref. [9], we denote such factors by $A_k(\sigma)$, k being the periodicity of the orbit. In the Hamiltonian limit and for $a = 6$ we have

$$\begin{aligned} S_6(\sigma) &= A_6^2(\sigma)\, B_6(\sigma)\, C_6(\sigma), && (1)\\ S_7(\sigma) &= A_7^2(\sigma)\, B_7(\sigma), && (2)\\ S_8(\sigma) &= A_8^2(\sigma)\, B_8(\sigma)\, C_8(\sigma), && (3)\end{aligned}$$

where

$$\begin{aligned} A_6(\sigma) &= \sigma - 2, && (4)\\ A_7(\sigma) &= \sigma^2 - 2\,\sigma - 6, && (5)\\ A_8(\sigma) &= \sigma^6 - 4\,\sigma^5 - 52\,\sigma^4 + 208\,\sigma^3 \\ &\quad +672\,\sigma^2 - 2688\,\sigma - 16, && (6)\end{aligned}$$

and all other coefficients, not needed here, are given in Ref. [9].

Therefore, chiral orbits exist for the factors ...

A. Period 6

The orbital equation for the simplest case of Eq. (4), $\sigma = 2$, is

$$O_1(x) = x^6 - 2x^5 - 14x^4 + 22x^3 + 62x^2 - 60x - 83, \quad (7)$$

which decomposes into $O_1(x) = f_1(x)\, f_2(x)$, where

$$\begin{aligned} f_1(x) &= x^3 - (1+\sqrt{3})x^2 - 6\,x + 5 + 6\,\sqrt{3}, && (8)\\ f_2(x) &= x^3 - (1-\sqrt{3})x^2 - 6\,x + 5 - 6\,\sqrt{3}. && (9)\end{aligned}$$

The zeros of f_1 are $x_1 = -2.409$, $x_2 = 2.101$, $x_3 = 3.040$, while those of f_2 are $x_4 = -2.317$, $x_5 = -0.926$, $x_6 = 2.511$. These orbital points produce two rather different orbits, namely

$$\begin{pmatrix} x_1 & x_6 & x_2 & x_5 & x_3 & x_4 \\ x_4 & x_1 & x_6 & x_2 & x_5 & x_3 \end{pmatrix}. \quad (10)$$

and

$$\begin{pmatrix} x_1 & x_4 & x_3 & x_5 & x_2 & x_6 \\ x_6 & x_1 & x_4 & x_3 & x_5 & x_2 \end{pmatrix}. \quad (11)$$

1. *Orbitas para $B_6 = 0$*

2. *Orbitas para $C_6 = 0$*

B. Period 7

For period-7 there are 18 possible values for the sum of orbital points. In this case $S_7(\sigma) = A_7(\sigma)^2\, B_7(\sigma)$ where

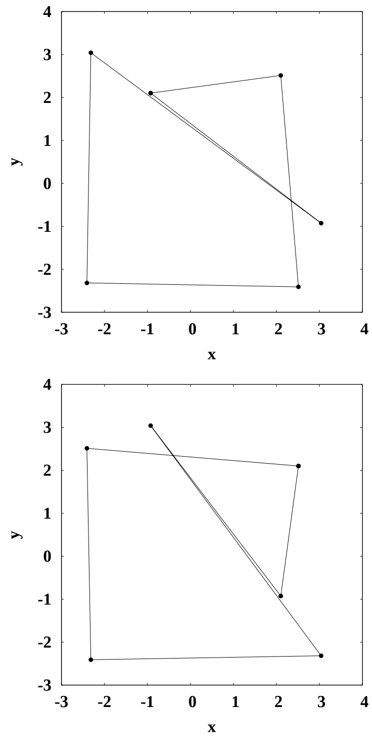

FIG. 1: The two pairs of period-6 chiral orbits $A_6 = 0$.

$$\begin{aligned} A_7(\sigma) &= \sigma^2 - 2\,\sigma - 6, && (12)\\ B_7(\sigma) &= \sigma^{14} + 2\,\sigma^{13} - 406\,\sigma^{12} + 288\,\sigma^{11} + 58540\,\sigma^{10} \\ &\quad -136808\,\sigma^9 - 3708984\,\sigma^8 + 11996864\,\sigma^7 \\ &\quad +107208320\,\sigma^6 - 411276032\,\sigma^5 \\ &\quad -1181332992\,\sigma^4 + 5368452864\,\sigma^3 \\ &\quad +901791744\,\sigma^2 - 11341783040\,\sigma \\ &\quad -3936915584. && (13)\end{aligned}$$

For $A_7(\sigma)$, the orbital equation due to $\sigma = 1 + \sqrt{7}$ is

$$\begin{aligned} P_7(x) &= x^7 - (1+\sqrt{7})x^6 - 2\,(8-\sqrt{7})x^5 \\ &\quad +(6+14\,\sqrt{7})x^4 + (87-22\,\sqrt{7})x^3 \\ &\quad +(21-61\,\sqrt{7})x^2 - 20\,(8-3\,\sqrt{7})x \\ &\quad -125 + 76\,\sqrt{7}, && (14)\end{aligned}$$

while its conjugate, for $\sigma = 1 - \sqrt{7}$, is easily obtained from $P_7(x)$ by replacing $\pm\sqrt{7} \to \mp\sqrt{7}$.

Equação orbital para $\sigma = 1 + \sqrt{7}$

$$\begin{aligned} P_1(x) &= x^7 - (1+\sqrt{7})x^6 - 2\,(8-\sqrt{7})x^5 \\ &\quad +2\,(3+7\,\sqrt{7})x^4 + (87-22\,\sqrt{7})x^3 \\ &\quad +(21-61\,\sqrt{7})x^2 - 20\,(8-3\,\sqrt{7})x \\ &\quad -125 + 76\,\sqrt{7} && (15)\end{aligned}$$

Soluções:

$x_1 = -2.363529$, $x_2 = -2.313389$, $x_3 = -0.7573026$, $x_4 = 0.9263635$, $x_5 = 2.414733$, $x_6 = 2.727116$, $x_7 = 3.011758$

$$\begin{pmatrix} x_1 & x_6 & x_4 & x_5 & x_3 & x_7 & x_2 \\ x_2 & x_1 & x_6 & x_4 & x_5 & x_3 & x_7 \end{pmatrix}. \quad (16)$$

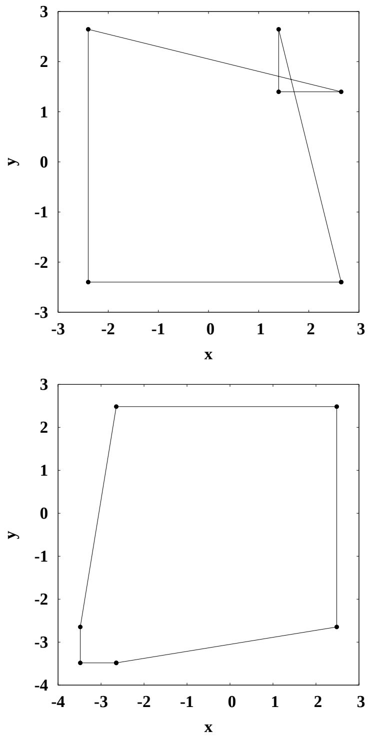

FIG. 2: The two pairs of period-6 orbits related with $B_6 = 0$.

$$\begin{pmatrix} x_1 & x_2 & x_7 & x_3 & x_5 & x_4 & x_6 \\ x_6 & x_1 & x_2 & x_7 & x_3 & x_5 & x_4 \end{pmatrix}. \tag{17}$$

Equação orbital para $\sigma = 1 - \sqrt{7}$

$$\begin{aligned} P_2(x) &= x^7 - (1-\sqrt{7})x^6 - 2\,(8+\sqrt{7})x^5 \\ &\quad +2\,(3-7\,\sqrt{7})x^4 + (22\,\sqrt{7}+87)x^3 \\ &\quad +(21+61\,\sqrt{7})x^2 - 20\,(8+3\,\sqrt{7})x \\ &\quad -125 - 76\,\sqrt{7} \end{aligned} \tag{18}$$

Soluções: $x_1 = -3.332654$ $x_2 = -2.603895$ $x_3 = -2.502687$ $x_4 = -0.9173544$ $x_5 = 2.089252$ $x_6 = 2.552380$ $x_7 = 3.069208$

$$\begin{pmatrix} x_1 & x_3 & x_7 & x_4 & x_5 & x_6 & x_2 \\ x_2 & x_1 & x_3 & x_7 & x_4 & x_5 & x_6 \end{pmatrix}. \tag{19}$$

$$\begin{pmatrix} x_1 & x_2 & x_6 & x_5 & x_4 & x_7 & x_3 \\ x_3 & x_1 & x_2 & x_6 & x_5 & x_4 & x_7 \end{pmatrix}. \tag{20}$$

C. Period 8

For period-8 there are 30 possible values for the sum of orbital points. Now we find $S_8(\sigma) = A_8^2(\sigma)\,B_8(\sigma)\,C_8(\sigma)$ where

$$A_8(\sigma) = \sigma^6 - 4\,\sigma^5 - 52\,\sigma^4 + 208\,\sigma^3$$

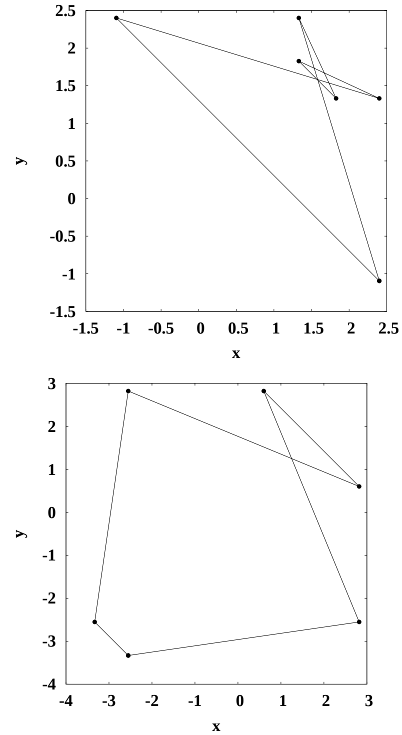

FIG. 3: Orbits of period-6 for $C_6 = 0$ (factor$\sigma^2 - 6\,\sigma - 18 = 0$).

$$\begin{aligned} &\quad +672\,\sigma^2 - 2688\,\sigma - 16, && (21) \\ B_8(\sigma) &= \sigma^6 + 8\,\sigma^5 - 208\,\sigma^4 - 896\,\sigma^3 \\ &\quad +12288\,\sigma^2 + 12288\,\sigma - 148480, && (22) \\ C_8(\sigma) &= \sigma^{12} - 480\,\sigma^{10} + 1792\,\sigma^9 + 64608\,\sigma^8 \\ &\quad -343296\,\sigma^7 - 2703200\,\sigma^6 + 16598016\,\sigma^5 \\ &\quad +4347648\,\sigma^4 - 18550272\,\sigma^3 \\ &\quad -8025600\,\sigma^2 + 995328\,\sigma + 553216. && (23) \end{aligned}$$

Roots of $A_8(\sigma)$:

$$-5.299, -4.889, -0.00594, 4.0444, 4.811, 5.338.$$

As órbitas relacionadas com estas raízes encontram-se na tabela I.

JACG thanks financial support from the Sonderforschungsbereich 404 of the DFG, Germany. Both authors thank Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil, for support.

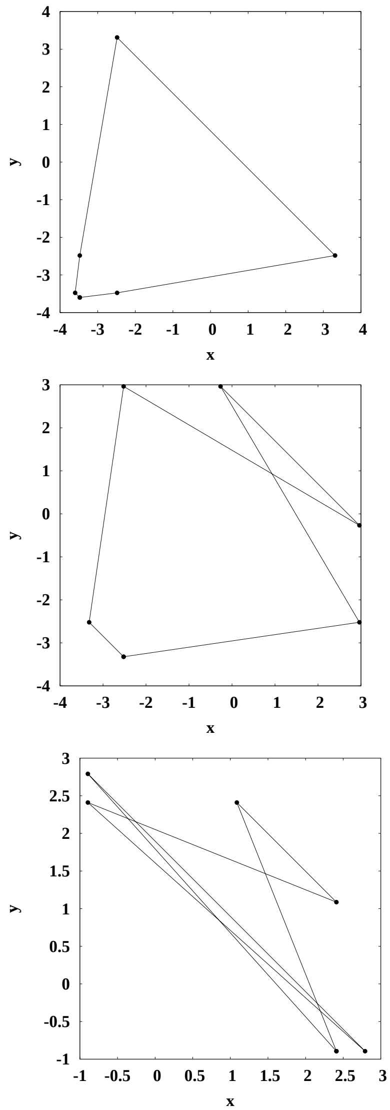

FIG. 4: Orbits of period-6 for $C_6 = 0$ (factor $\sigma^3 + 8\,\sigma^2 - 70\,\sigma - 228 = 0$).

[1] I. Agranat, H. Caner and J. Caldwell, Nature Reviews Drug Discovery **1**, 753-768 (2002).
[2] H.G. Kausch, hep-th/9209093.
[3] A.A. Belavin, A.M. Poliakov and A.B. Zamolodchikov, Nucl. Phys. B241, 333 (1984).
[4] A. Cappelli, C. Itzykson ans J.-B. Zuber, Commun. Mat. Phys., **113**, 1 (1987).
[5] V. Drinfeld, Amer. J. Math. **10**, 85-114 (1983).
[6] H. Kawamura PRL **68** 3785 (1992); PRL **80** 5421 (1998); .....
[7] G. Tatara and H. Kawamura, J. Phys. Soc. Japan **71**, 2613 (2002). H. Kawamura, Phys. Rev. Lett. **90**, 47202 (2003).
[8] P.Pureur, F. Wolff Fabri, J. Schaf and I. A. Campbell, Europhys. Lett. **67**, 123 (2004).
[9] A. Endler and J.A.C. Gallas, *Rational reductions in sums of orbital coordinates for a Hamiltonian repeller*, preprint, 2005.

O	σ	x_i	y_i
1	4.0444625274632178345	−2.4136818898204280512	2.5061371791343513022
2		−2.4136818898204280512	−2.3319974443794833734
3	4.8119429971428316647	−2.4249340808746823702	−2.3839225073112408926
4		−2.4249340808746823702	2.5036172107166258738
5	5.3383419167222111074	−2.3796082438653275681	−2.3144711608338244915
6		−2.3796082438653275681	2.6519357665619015952
7	−.0059435657301925738354	−3.3269320848103024024	−2.5612116747899661454
8		−3.3269320848103024024	−2.5072654221502590292
9	−4.8897858305569149683	−3.3280553369341453824	−2.6157097269493868247
10		−3.3280553369341453824	−2.4602425987429100130
11	−5.2990180450411530644	−3.4778667307895246147	−3.4649778907246866276
12		−3.4778667307895246147	−2.6305791064082163332

TABLE I: Tabela de órbitas chirais para período 8, órbitas relacionadas com $A_8(\sigma) = 0$.