Chiral orbits

Antônio Endler[1,2,*] and Jason A.C. Gallas[1,2,†]

[1] *Institut für Computerphysik, Universität Stuttgart, D-70569 Stuttgart, Germany*

[2] *Instituto de Física, Universidade Federal do Rio Grande do Sul, 91501-970 Porto Alegre, Brazil*

(Dated: Draft of January 12, 2005)

We define and analyse "chiral orbits".

I. INTRODUCTION

II. CHIRAL ORBITS

Chiral orbits appear when there is a factor with multiplicity higher than 1 in $S_k(\sigma)$. Following Ref. [1], we denote such factors by $A_k(\sigma)$, k being the periodicity of the orbit. In the Hamiltonian limit and for $a=6$ we have

$$\begin{aligned} S_6(\sigma) &= A_6^2(\sigma)\, B_6(\sigma)\, C_6(\sigma), & (1)\\ S_7(\sigma) &= A_7^2(\sigma)\, B_7(\sigma), & (2)\\ S_8(\sigma) &= A_8^2(\sigma)\, B_8(\sigma)\, C_8(\sigma), & (3)\end{aligned}$$

where

$$\begin{aligned} A_6(\sigma) &= \sigma - 2, & (4)\\ A_7(\sigma) &= \sigma^2 - 2\,\sigma - 6, & (5)\\ A_8(\sigma) &= \sigma^6 - 4\,\sigma^5 - 52\,\sigma^4 + 208\,\sigma^3 \\ &\quad +672\,\sigma^2 - 2688\,\sigma - 16, & (6)\end{aligned}$$

and all other coefficients, not needed here, are given in Ref. [1].

Therefore, chiral orbits exist for the factors ...

A. Period 6

The orbital equation for the simplest case of Eq. (4), $\sigma = 2$, is

$$O_1(x) = x^6 - 2x^5 - 14x^4 + 22x^3 + 62x^2 - 60x - 83, \quad (7)$$

which decomposes into $O_1(x) = f_1(x)\, f_2(x)$, where

$$\begin{aligned} f_1(x) &= x^3 - (1+\sqrt{3})x^2 - 6\,x + 5 + 6\,\sqrt{3}, & (8)\\ f_2(x) &= x^3 - (1-\sqrt{3})x^2 - 6\,x + 5 - 6\,\sqrt{3}. & (9)\end{aligned}$$

The zeros of f_1 are $x_1 = -2.409$, $x_2 = 2.101$, $x_3 = 3.040$, while those of f_2 are $x_4 = -2.317$, $x_5 = -0.926$, $x_6 = 2.511$. These orbital points produce two rather different orbits, namely

$$\begin{pmatrix} x_1 & x_6 & x_2 & x_5 & x_3 & x_4 \\ x_4 & x_1 & x_6 & x_2 & x_5 & x_3 \end{pmatrix}. \quad (10)$$

and

$$\begin{pmatrix} x_1 & x_4 & x_3 & x_5 & x_2 & x_6 \\ x_6 & x_1 & x_4 & x_3 & x_5 & x_2 \end{pmatrix}. \quad (11)$$

B. Period 7

For period-7 there are 18 possible values for the sum of orbital points. In this case $S_7(\sigma) = A_7(\sigma)^2\, B_7(\sigma)$ where

$$\begin{aligned} A_7(\sigma) &= \sigma^2 - 2\,\sigma - 6, & (12)\\ B_7(\sigma) &= \sigma^{14} + 2\,\sigma^{13} - 406\,\sigma^{12} + 288\,\sigma^{11} + 58540\,\sigma^{10} \\ &\quad -136808\,\sigma^9 - 3708984\,\sigma^8 + 11996864\,\sigma^7 \\ &\quad +107208320\,\sigma^6 - 411276032\,\sigma^5 \\ &\quad -1181332992\,\sigma^4 + 5368452864\,\sigma^3 \\ &\quad +901791744\,\sigma^2 - 11341783040\,\sigma \\ &\quad -3936915584. & (13)\end{aligned}$$

For $A_7(\sigma)$, the orbital equation due to $\sigma = 1+\sqrt{7}$ is

$$\begin{aligned} P_7(x) &= x^7 - (1+\sqrt{7})x^6 - 2\,(8-\sqrt{7})x^5 \\ &\quad +(6+14\,\sqrt{7})x^4 + (87-22\,\sqrt{7})x^3 \\ &\quad +(21-61\,\sqrt{7})x^2 - 20\,(8-3\,\sqrt{7})x \\ &\quad -125+76\,\sqrt{7}, & (14)\end{aligned}$$

while its conjugate, for $\sigma = 1-\sqrt{7}$, is easily obtained from $P_7(x)$ by replacing $\pm\sqrt{7} \to \mp\sqrt{7}$.

Equação orbital para $\sigma = 1+\sqrt{7}$

$$\begin{aligned} P_1(x) &= x^7 - (1+\sqrt{7})x^6 - 2\,(8-\sqrt{7})x^5 \\ &\quad +2\,(3+7\,\sqrt{7})x^4 + (87-22\,\sqrt{7})x^3 \\ &\quad +(21-61\,\sqrt{7})x^2 - 20\,(8-3\,\sqrt{7})x \\ &\quad -125+76\,\sqrt{7} & (15)\end{aligned}$$

Soluções:

$x_1 = -2.363529$, $x_2 = -2.313389$, $x_3 = -0.7573026$, $x_4 = 0.9263635$, $x_5 = 2.414733$, $x_6 = 2.727116$, $x_7 = 3.011758$

$$\begin{pmatrix} x_1 & x_6 & x_4 & x_5 & x_3 & x_7 & x_2 \\ x_2 & x_1 & x_6 & x_4 & x_5 & x_3 & x_7 \end{pmatrix}. \quad (16)$$

* E-mail: aendler@if.ufrgs.br

† URL: http://www.ica1.uni-stuttgart.de/~jgallas

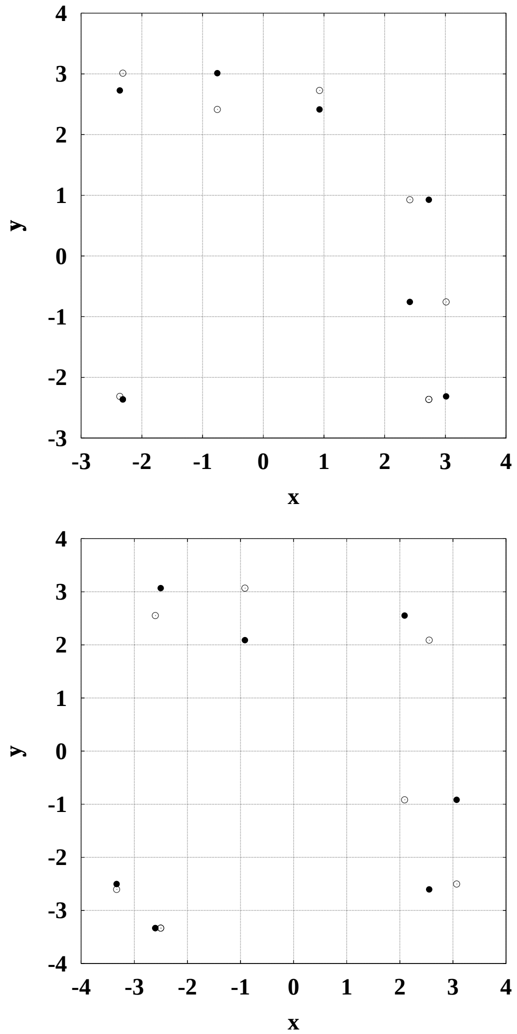

FIG. 1: The two pairs of period-7 chiral orbits.

$$\begin{pmatrix} x_1 & x_2 & x_7 & x_3 & x_5 & x_4 & x_6 \\ x_6 & x_1 & x_2 & x_7 & x_3 & x_5 & x_4 \end{pmatrix}. \qquad (17)$$

Equação orbital para $\sigma = 1 - \sqrt{7}$

$$\begin{aligned} P_2(x) \;=\; & x^7 - (1-\sqrt{7})x^6 - 2\,(8+\sqrt{7})x^5 \\ & +2\,(3-7\sqrt{7})x^4 + (22\sqrt{7}+87)x^3 \\ & +(21+61\sqrt{7})x^2 - 20\,(8+3\sqrt{7})x \\ & -125 - 76\sqrt{7} \end{aligned} \qquad (18)$$

Soluções: $x_1 = -3.332654$ $x_2 = -2.603895$ $x_3 = -2.502687$ $x_4 = -0.9173544$ $x_5 = 2.089252$ $x_6 = 2.552380$ $x_7 = 3.069208$

$$\begin{pmatrix} x_1 & x_3 & x_7 & x_4 & x_5 & x_6 & x_2 \\ x_2 & x_1 & x_3 & x_7 & x_4 & x_5 & x_6 \end{pmatrix}. \qquad (19)$$

$$\begin{pmatrix} x_1 & x_2 & x_6 & x_5 & x_4 & x_7 & x_3 \\ x_3 & x_1 & x_2 & x_6 & x_5 & x_4 & x_7 \end{pmatrix}. \qquad (20)$$

C. Period 8

For period-8 there are 30 possible values for the sum of orbital points. Now we find $S_8(\sigma) = A_8^2(\sigma)\,B_8(\sigma)\,C_8(\sigma)$ where

$$\begin{aligned} A_8(\sigma) \;=\; & \sigma^6 - 4\,\sigma^5 - 52\,\sigma^4 + 208\,\sigma^3 \\ & +672\,\sigma^2 - 2688\,\sigma - 16, && (21) \\ B_8(\sigma) \;=\; & \sigma^6 + 8\,\sigma^5 - 208\,\sigma^4 - 896\,\sigma^3 \\ & +12288\,\sigma^2 + 12288\,\sigma - 148480, && (22) \\ C_8(\sigma) \;=\; & \sigma^{12} - 480\,\sigma^{10} + 1792\,\sigma^9 + 64608\,\sigma^8 \\ & -343296\,\sigma^7 - 2703200\,\sigma^6 + 16598016\,\sigma^5 \\ & +4347648\,\sigma^4 - 18550272\,\sigma^3 \\ & -8025600\,\sigma^2 + 995328\,\sigma + 553216. && (23) \end{aligned}$$

Roots of $A_8(\sigma)$:

$$-5.299, -4.889, -0.00594, 4.0444, 4.811, 5.338.$$

JACG thanks financial support from the Sonderforschungsbereich 404 of the DFG, Germany. Both authors thank Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil, for support.

[1] A. Endler and J.A.C. Gallas, *Rational reductions in sums of orbital coordinates for a Hamiltonian repeller*, preprint, 2005.