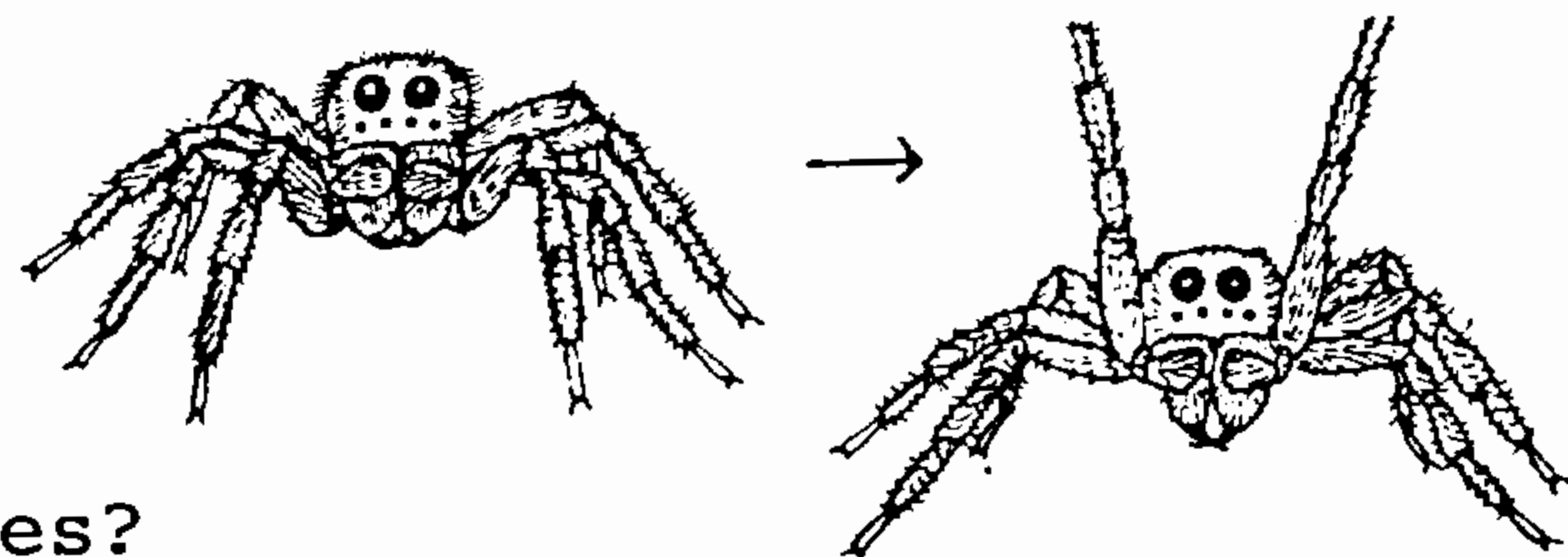


## 1. INTRODUCTION



What are long-leggety beasties?

Long-leggety beasties are to be seen in any field theory or statistical mechanics textbook; they are Feynman diagrams, Green functions, S-matrix elements, correlation functions, and so on. They represent sums of probabilities (statistical mechanics) or probability amplitudes (quantum mechanics).

There are two ways of visualizing long-leggety beasties<sup>†</sup>.

In the first picture the transition probability (amplitude) is the sum of all ways in which particles can propagate, disintegrate and recombine before reaching a detector. Each possibility is represented by a Feynman diagram, and the penalty associated with each choice is given by a Feynman integral.

In the second picture the transition probability (amplitude) is a sum over all "paths" which the system can take between the initial and the final state. The penalty to be paid for a particular path is assessed by a Boltzmann factor (phase factor). A process is dominated by the classical paths, and the fluctuation (quantum) effects arise from the heavily penalized deviations away from the beaten path.

The two pictures are equivalent. The second (path integrals) is a "Fourier" transform of the first (generating functionals). In some contexts, such as in perturbative calculations, generating functionals are the practical choice. In others, such as in identifying the dominant classical configurations, or in exploiting symmetries of a theory, the path integral formulation might be more suggestive.

In these notes we put the usual logic of field theory textbooks on its head; we start with the Feynman rules and end with Lagrangians. We find it easier to understand field theory this way: for many particle physicists, diagrams are an important tool for developing field-theoretic intuition.

Our attitude will be eclectic. We shall start by building up generating functionals using vertices and propagators as

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<sup>†</sup>R. Herrick has in his poem "On Julia's Legs" suggested a third way: "Fain would I kiss my Julia's dainty leg, which is as white and hairless as an egg".

simple building blocks. Then we shall rewrite the results in terms of path integrals, and from then on use either formalism, whichever may be more expedient. Each particular physical theory brings in its own set of ailments (ultraviolet divergences, ill-defined path integrals, etc.), but the general formalism should be good enough to describe anything under the sun, from statistical mechanics to lattice gauge theories to continuum theories to gravity and cosmology. The general formalism is straightforward and intuitive. The real work starts only with specialization to a particular theory; the dominant classical configurations have to be identified, divergent sums (integrals) regularized, etc.

We will apply the general formalism to QCD. Chapter 6 is a rehash of Benny Lautrup's "Ghoulies and Ghosties". This construction yields QCD Feynman rules and bare Ward identities. In chapter 7 we feed these into the general formalism to obtain the Ward identities for full Green functions. At this point our patience runs out, and the proof of renormalizability of QCD and the evaluation of the running coupling constants, scaling violations and hadron masses are left as exercises for the reader.

I have included much graphic gore in these notes. The reason is that I fear that the perturbation theory is here to stay; it will not go away even if the gauge theories do. At least, if I ever have to do a perturbative calculation again, I will know where to look up the diagrams. The reader is advised to skip over lengthy perturbative expansions - most particle physicists reach tenure without doing anything more strenuous than one-loop Feynman integrals. The exercises are another matter - we have relegated much of the conceptually dull but technically important material to the exercises. They are of three kinds: trivial, undoable, and wrong.

There is nothing in these lectures that is not well-known and has not been published many other places. The only excuse for writing them up is that they seem to resemble no other field theory text on the market. It cannot be precluded that that might be considered a virtue.

## A. Land of Quefithe

Once (and it was not yesterday) there lived a very young mole and a very young crow who, having heard of the fabulous land called Quefithe, decided to visit it. Before starting out, they went to the wise owl and asked what Quefithe was like.

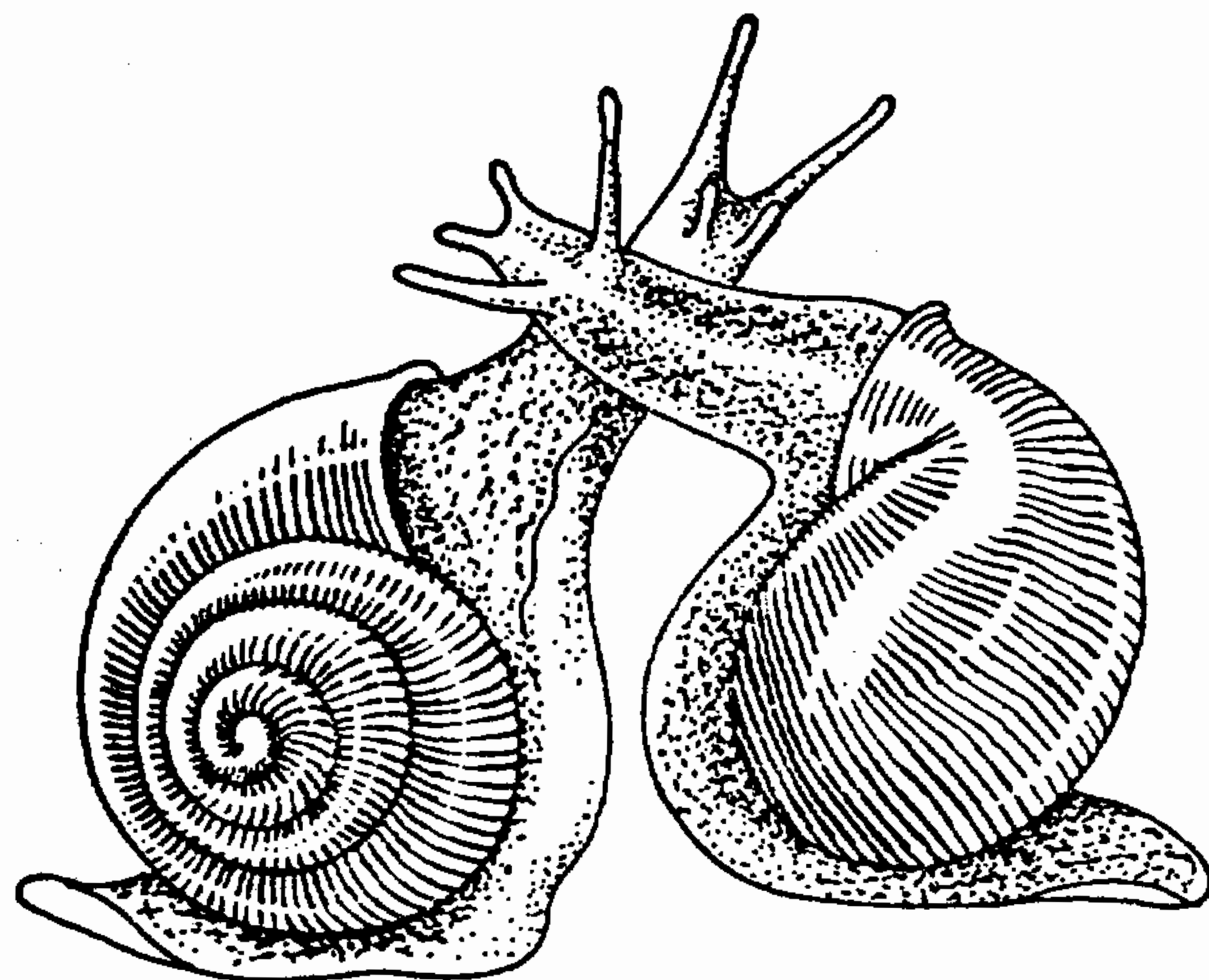
Owl's description of Quefithe was quite confusing. He said that in Quefithe everything was both up and down. If you knew where you were, there was no way of knowing where you were going, and conversely, if you knew where you were going, there was no way of knowing where you were. The young mole and the young crow did not understand much, so they went instead to the old eagle and asked him what Quefithe was like. The eagle shook his white-feathered head, sized them up with his fierce eyes, and said: "Action gives automatically invariant description of Quefithe. You must study the unitary representations of the Lorentz group". The mole and the crow waited for more, but the eagle remained silent, his gaze fixed on an unfathomable string in the sky.

Clearly, if they were ever going to learn anything about Quefithe, they had to see it for themselves. And that is what they did.

After a few years had passed, the mole came back. He said that Quefithe consisted of lots of tunnels. One entered a hole and wandered through a maze, tunnels splitting and rejoining, until one found the next hole and got out. Quefithe sounded like a place only a mole would like, and nobody wanted to hear more about it.

Not much later the crow landed, flapping its wings and crowing excitedly. Quefithe was amazing, it said. The most beautiful landscape with high mountains, perilous passes and deep valleys. The valley floors were teeming with little moles who were scurrying down rutted paths. The crow sounded like he had taken too many bubble baths, and many who heard him shook their heads. The frogs kept on croaking "it is not rigorous, it is not rigorous!" The eagle said: "It is frightful nonsense. One must study the unitary representations of the Lorentz group". But there was something about crow's enthusiasm that was infectious.

The most puzzling thing about it all was that the mole's description of Quefithe sounded nothing like the crow's description. Some even doubted that the mole and the crow had ever gotten to the mythical land. Only the fox, who was by nature very curious, kept running back and forth between the mole and the crow and asking questions, until he was sure that he understood them both. Nowadays, anybody can get to Quefithe - even snails.



*two hermaphroditic snails.*